WASHINGTON OBSERVATIONS FOR 1870.—APPENDIX I.

REPORT

ON THE

DIFFERENCE OF LONGITUDE

BETWEEN

WASHINGTON AND ST. LOUIS.

BY

WILLIAM HARKNESS,

PROFESSOR OF MATHEMATICS, U. S. NAVY.

PREPARED AT THE U. S. NAVAL OBSERVATORY

BY ORDER OF

REAR-ADMIRAL B. F. SANDS, U. S. N.,

SUPERINTENDENT.

WASHINGTON:
GOVERNMENT PRINTING OFFICE.
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REPORT

ON THE

DIFFERENCE OF LONGITUDE

BETWEEN

WASHINGTON AND ST. LOUIS.

UNITED STATES NAVAL OBSERVATORY,
Washington, November 14, 1872.

SIR: I have the honor to submit to you the following report on the determination of the difference of longitude between Washington and St. Louis, of which you directed me to take charge so far as this Observatory is concerned.

I.—INTRODUCTORY.

The operations described in this report were initiated by the United States Coast Survey, and the Observatory took part in them at the request of that institution, with the understanding that the observations at St. Louis should be made by Coast Survey officers, and those at Washington by Observatory officers; and that at the conclusion of the campaign complete copies of the observations and reductions should be exchanged for each other's use. The observations here were made by myself and Assistant Observer Edgar Frisby, and reduced entirely by me. The observations at St. Louis were made by Professor William Eimbeck, of the Coast Survey, and reduced by Professor R. Keith, of the Coast Survey; but, as the right ascensions which the latter gentleman adopted for some of the stars employed differed slightly from those used at this Observatory, before his work could be compared with my own it required a few small corrections, which have been introduced by Mr. Frisby and myself. The arrangements for the use of the Western Union Telegraph Company's lines between Washington and St. Louis were made by the officers of the Coast Survey, but I cannot refrain from expressing my thanks to Mr. M. Marean, the Western Union Company's electrical superintendent in this city, for his kindness in promptly making the necessary connections between the different wires at the main office here.

II.—DESCRIPTION OF OBSERVING-STATIONS.

The observing-station at Washington was the present site of the transit circle, which is 77.8 feet due west of the center of the dome of the Observatory. Its geographical position is:

Latitude, ... $+ 38^\circ\ 53'\ 38''.8$
Longitude, west of Greenwich, . $5^h\ 8^m\ 12^s.0$

The station at St. Louis was in a small observatory erected on St. Charles street, between Seventeenth and Eighteenth streets, in the southwest corner of the Washington University grounds. These grounds, rectangular in form, are bounded on the north

by Washington avenue, and on the south by St. Charles street, occupying the whole space between these streets, which is 150 feet. They are bounded on the east by Seventeenth street, and extend 206 feet 10½ inches toward Eighteenth street. Washington avenue and St. Charles streets are parallel to each other, and run in the direction south 75° east. Seventeenth and Eighteenth streets are also parallel to each other and run in the direction south 15° west. The small observatory building measured eight feet from north to south, and ten feet from east to west. It contained two piers, the transit instrument being mounted on the eastern one and the zenith telescope on the western one. The following distances were measured from the station point on the transit pier, viz: to the western boundary of the university grounds, 13 feet 5 inches; to the line of curb-stones on the northern side of St. Charles street, 14 feet 4 inches; and to the produced western face of the Scientific Department building, 10 feet 4 inches. The approximate latitude of the station was $+ 38° 37'$.

III.—INSTRUMENTS EMPLOYED AT WASHINGTON.

The Transit Circle, whose object-glass has a focal length of 145 inches, and a clear aperture of 8.52 inches, was used with an eye-piece which produced a magnifying-power of 186 diameters. Throughout the whole series of observations the clamp end of the axis was to the east. A description of this instrument is given in Appendix I to the Washington Observations for 1865.

The Kessels Sidereal Clock, No. 1324, which is the Observatory standard, was employed in connection with

A Chronograph, having a barrel 8.15 inches in diameter and 24.0 inches long, revolving once each minute. This chronograph will run continuously for four hours without requiring the paper to be changed. It has but a single pen, with which both the clock-signals and those made by the observer are recorded.

The Electro-magnetic Apparatus, which was used for sending and receiving the longitude-signals is entirely automatic. It would occupy too much space to explain it here, but a full description may be found in Appendix I to the Washington Observations for 1867.

IV.—INSTRUMENTS EMPLOYED AT ST. LOUIS.

United States Coast Survey Portable Transit Instrument No. 7, made by William Würdemann, of Washington. Its object-glass has a focal distance of 25¾ inches, and a clear aperture of 2.03 inches. A diagonal eye-piece was used, which produced a magnifying-power of 67 diameters. The length of the axis between the Y's is 14 inches, and the pivots are 0.71 of an inch in diameter. It is provided with two finding-circles, each 4 inches in diameter, graduated to every 20′, and reading by means of two verniers to single minutes.

A Sidereal Box Chronometer, Kessels and Dent No. 1287.

A Mean Time Box Chronometer, Dent No. 2748.

The Electro-magnetic Apparatus employed was the ordinary Morse receiving magnet, sounder, and key, together with a break-circuit key; all of which were in the Western Union Telegraph Office in the Merchant's Exchange, on First street, between Walnut and Market streets.

V.—METHOD OF REDUCING THE OBSERVATIONS FOR TIME.

Let

$\alpha =$ apparent right ascension of the star observed;

$T' =$ observed clock or chronometer time of star's transit;

$T_0 =$ time, by face of clock or chronometer, for which clock or chronometer correction is to be determined;

$R =$ rate per hour of clock or chronometer;

$\varDelta T_0 =$ correction of clock or chronometer at the instant when its face indicated the time T_0;

$v =$ effect of errors of observation;

a, b, and $c =$ respectively, the azimuth, level, and collimation constants;

A, B, and $C =$ respectively, the azimuth, level, and collimation factors,

Then each star observed will furnish one equation of condition of the form

$$0 = \alpha - [\, T' + \varDelta T_0 + R\,(T' - T_0) + Aa + Bb + Cc \,] - v$$

and from all the equations thus obtained the most probable values of the quantities considered as unknown can be found by the method of least squares.

When the observations have been made with a fixed instrument, the quantities sought are usually $\varDelta T_0$ and R; but if a portable instrument has been used they will generally be $\varDelta T_0$, R, and a. Sometimes it is convenient to make T_0 equal to the mean of the observed times of transit of all the stars in the group, and then R is found from the difference between the values of $\varDelta T_0$ given by two groups of stars separated by an interval of some hours. It is always advantageous to have the unknown quantities in the equations of condition quite small, and therefore, when possible, it is best to introduce closely approximate values of these quantities, and to solve only for small corrections to these approximate values. Thus, if R is large, and r is an approximate value of it, we write $R = \delta r + r$, and, substituting that value in the equations of condition, we determine δr. In the same manner, if $\varDelta T_0$ is large, and θ is an approximate value of it, we write $\varDelta T_0 = \theta + \delta\theta$, and, substituting that value in the equations of condition, we determine $\delta\theta$.

The factors A, B, and C may be computed by means of the formulæ

$$A = \sin\,(\varphi - \delta)\,\sec\delta = \sin\varphi - \cos\varphi\tan\delta$$
$$B = \cos\,(\varphi - \delta)\,\sec\delta = \cos\varphi + \sin\varphi\tan\delta$$
$$C = \sec\delta$$

in which φ is the latitude of the place of observation, and δ the declination of the star observed. For a culmination below the pole, $180° - \delta$ must be substituted instead of δ. The following rules are sometimes convenient for determining the signs of these quantities.

A is positive, except for stars between the zenith and the pole.

B and C are positive, except for stars below the pole.

Table I gives the adopted mean places of all the stars employed in the longitude operations, together with the corresponding values of A, B, and C, both at Washington and St. Louis.

Table I.—*Adopted Mean Right Ascensions for* 1870.0 *of Stars used in the Determination of the Difference of Longitude between Washington and St. Louis; together with the Constants A, B, and C for Azimuth, Level, and Collimation at each Station.*

Name of Star.	Right Ascension.	Washington.			St. Louis.		
		A	*B*	*C*	*A*	*B*	*C*
	h. m. s.						
ε Hydræ	8 39 53.45	. .	. .	. .	+ 0.529	+ 0.857	+ 1.01
ι Ursæ Majoris	8 50 17.72	. .	. .	. .	− 0.247	+ 1.490	+ 1.52
κ Cancri	9 0 42.25	. .	. .	. .	+ 0.462	+ 0.918	+ 1.03
1 Draconis	9 18 20.13	. .	. .	. .	− 4.85	+ 5.15	+ 7.08
α Hydræ	9 21 11.96	. .	. .	. .	+ 0.734	+ 0.693	+ 1.01
β Cephei, S. P.	9 26 58.46	. .	. .	. .	+ 2.76	− 0.939	− 2.92
ε Leonis	9 38 28.10	. .	. .	. .	+ 0.267	+ 1.06	+ 1.10
μ Leonis	9 45 21.94	. .	. .	. .	+ 0.232	+ 1.09	+ 1.12
79 Draconis, S. P.	9 51 15.02	. .	. .	. .	+ 3.19	− 1.27	− 3.44
α Leonis	10 1 26.82	. .	. .	. .	+ 0.449	+ 0.920	+ 1.02
32 Ursæ Majoris	10 8 33.82	. .	. .	. .	− 1.11	+ 2.17	+ 2.44
γ^1 Leonis	10 12 48.14	. .	. .	. .	+ 0.331	+ 1.02	+ 1.07
ϑ Draconis	10 23 58.40	. .	. .	. .	− 2.60	+ 3.36	+ 4.25
l Leonis	10 42 25.37	. .	. .	. .	+ 0.469	+ 0.904	+ 1.02
ι Cephei, S. P.	10 45 3.53	. .	. .	. .	+ 2.34	− 0.589	− 2.41
α Ursæ Majoris	10 55 41.04	. .	. .	. .	− 0.875	+ 1.98	+ 2.16
δ Leonis	11 7 11.52	. .	. .	. .	+ 0.320	+ 1.02	+ 1.07
δ Crateris	11 12 50.58	. .	. .	. .	+ 0.820	+ 0.624	+ 1.03
τ Leonis	11 21 15.13	+ 0.579	+ 0.817	+ 1.00	+ 0.574	+ 0.820	+ 1.00
λ Draconis	11 23 39.55	. .	. .	. .	− 1.53	+ 2.50	+ 2.93
υ Leonis	11 30 17.60	+ 0.629	+ 0.777	+ 1.00			
β Leonis	11 42 25.66	+ 0.416	+ 0.949	+ 1.04	+ 0.411	+ 0.952	+ 1.04
γ Ursæ Majoris	11 46 58.98	. .	. .	. .	− 0.468	+ 1.65	+ 1.72
ο Virginis	11 58 35.22	+ 0.498	+ 0.883	+ 1.01	+ 0.493	+ 0.886	+ 1.01
4 Draconis	12 6 4.78	. .	. .	. .	− 3.16	+ 3.81	+ 4.95
η Virginis	12 13 15.35	+ 0.627	+ 0.779	+ 1.00	+ 0.622	+ 0.782	+ 1.00
β Corvi	12 27 33.74	+ 0.953	+ 0.515	+ 1.08			
κ Draconis	12 27 55.31	. .	. .	. .	− 1.58	+ 2.54	+ 3.00
12 Canum Venaticorum	12 49 56.63	− 0.002	+ 1.29	+ 1.29			
θ Virginis	13 3 13.27	+ 0.694	+ 0.725	+ 1.00			
Polaris, S. P.	13 11 17.34	+32.85	−25.22	− 41.42			
α Virginis	13 18 20.85	+ 0.772	+ 0.660	+ 1.02			
ζ Virginis	13 28 4.26	+ 0.626	+ 0.779	+ 1.00			
η Bootis	13 48 29.71	+ 0.360	+ 0.995	+ 1.06			
α Bootis	14 9 43.98	+ 0.347	+ 1.00	+ 1.06			

TABLE I.—*Adopted Mean Right Ascensions for* 1870.0, *&c.*—Continued.

Name of Star.	Right Ascension.	Washington.			St. Louis.		
		A	*B*	*C*	*A*	*B*	*C*
	h. m. s.						
ε Bootis	14 39 18.61	+ 0.219	+ 1.11	+ 1.13			
α^2 Libræ	14 43 41.42	+ 0.842	+ 0.606	+ 1.04	+ 0.838	+ 0.610	+ 1.04
β Ursæ Minoris	14 51 6.66	. .	. .	. .	− 2.13	+ 3.06	+ 3.79
β Bootis	14 57 2.89	− 0.046	+ 1.32	+ 1.32	− 0.052	+ 1.32	+ 1.32
β Libræ	15 10 0.87	+ 0.750	+ 0.679	+ 1.01	+ 0.746	+ 0.683	+ 1.01
μ^1 Bootis	15 19 34.84	+ 0.023	+ 1.27	+ 1.27	+ 0.017	+ 1.27	+ 1.27
γ^2 Ursæ Minoris	15 20 57.15	. .	. .	. .	− 1.82	+ 2.74	+ 3.29
α Coronæ Borealis	15 29 11.08	+ 0.229	+ 1.10	+ 1.12	+ 0.223	+ 1.10	+ 1.12
α Serpentis	15 37 51.99	+ 0.535	+ 0.853	+ 1.01	+ 0.530	+ 0.856	+ 1.01
ε Serpentis	15 44 20.27	+ 0.562	+ 0.832	+ 1.00	+ 0.557	+ 0.834	+ 1.00
ζ Ursæ Minoris	15 48 45.50	. .	. .	. .	− 3.12	+ 3.77	+ 4.89
ε Coronæ Borealis	15 52 12.44	. .	. .	. .	+ 0.222	+ 1.102	+ 1.12
β^1 Scorpii	15 57 52.89	. .	. .	. .	+ 0.899	+ 0.559	+ 1.06
Groombridge 2320	16 5 58.46	. .	. .	. .	− 1.33	+ 2.33	+ 2.69
δ Ophiuchi	16 7 32.12	. .	. .	. .	+ 0.676	+ 0.739	+ 1.00
τ Herculis	16 15 50.13	. .	. .	. .	− 0.189	+ 1.44	+ 1.46
η Draconis	16 22 14.18	. .	. .	. .	− 0.836	+ 1.94	+ 2.12
ζ Ophiuchi	16 30 0.16	. .	. .	. .	+ 0.764	+ 0.668	+ 1.02
η Herculis	16 38 26.43	− 0.005	+ 1.29	+ 1.29	− 0.011	+ 1.29	+ 1.29
κ Ophiuchi	16 51 30.97	+ 0.497	+ 0.884	+ 1.01	+ 0.492	+ 0.887	+ 1.01
d Herculis	16 56 48.28	+ 0.107	+ 1.20	+ 1.20	+ 0.101	+ 1.20	+ 1.20
ε Ursæ Minoris	16 59 22.74	− 5.09	+ 5.39	+ 7.42	− 5.12	+ 5.37	+ 7.42
α^1 Herculis	17 8 43.26	+ 0.426	+ 0.941	+ 1.03	+ 0.421	+ 0.944	+ 1.03
44 Ophiuchi	17 18 26.00	+ 0.974	+ 0.498	+ 1.09	+ 0.971	+ 0.502	+ 1.09
Groombridge 966, S. P.	17 22 21.68	. .	. .	. .	+ 3.53	− 1.54	− 3.85
α Ophiuchi	17 28 54.07	+ 0.453	+ 0.919	+ 1.02	+ 0.448	+ 0.922	+ 1.02
ω Draconis	17 37 42.96	. .	. .	. .	− 1.39	+ 2.39	+ 2.77
μ Herculis	17 41 22.32	+ 0.216	+ 1.11	+ 1.13			
γ Draconis	17 53 35.37	. .	. .	. .	− 3.60	+ 1.57	+ 1.61
γ^2 Sagittari	17 57 27.50	+ 1.08	+ 0.413	+ 1.16			
μ^1 Sagittarii	18 5 59.38	+ 0.926	+ 0.538	+ 1.07	+ 0.923	+ 0.542	+ 1.07
δ Ursæ Minoris	18 14 16.33	−12.47	+11.35	+ 16.86	−12.52	+11.28	+ 16.86
η Serpentis	18 14 35.03	. .	. .	. .	+ 0.663	+ 0.749	+ 1.00
1 Aquilæ	18 28 8.00	+ 0.742	+ 0.686	+ 1.01			
α Lyræ	18 32 32.26	+ 0.006	+ 1.28	+ 1.28			
51 Cephei, S. P.	18 38 43.17	+16.73	−12.21	− 20.72	+16.86	−12.18	− 20.72
β Lyræ	18 45 16.87	+ 0.117	+ 1.19	+ 1.19			
ζ Aquilæ	18 59 26.14	+ 0.438	+ 0.930	+ 1.03	+ 0.433	+ 0.933	+ 1.03
d Sagittarii	19 10 1.68	. .	. .	. .	+ 0.894	+ 0.564	+ 1.06
δ Aquilæ	19 18 56.64	+ 0.589	+ 0.809	+ 1.00	+ 0.584	+ 0.812	+ 1.00

TABLE I.—*Adopted Mean Right Ascensions for* 1870.0, *&c.*—Continued.

Name of Star.	Right Ascension.	Washington.			St. Louis.		
		A	*B*	*C*	*A*	*B*	*C*
	h. m. s.						
κ Aquilæ	19 29 53.85	+ 0.728	+ 0.697	+ 1.01	+ 0.724	+ 0.701	+ 1.01
γ Aquilæ	19 40 4.79	+ 0.486	+ 0.892	+ 1.02	+ 0.481	+ 0.895	+ 1.02
α Aquilæ	19 44 26.46	+ 0.511	+ 0.872	+ 1.01	+ 0.506	+ 0.875	+ 1.01
β Aquilæ	19 48 55.68	+ 0.545	+ 0.845	+ 1.01	+ 0.540	+ 0.848	+ 1.01
λ Ursæ Minoris . . .	19 54 16.90	−40.52	+33.99	+ 52.89	−40.68	+33.78	+ 52.89
α² Capricorni	20 10 50.44	+ 0.806	+ 0.634	+ 1.03	+ 0.802	+ 0.638	+ 1.03
ε Delphini	20 27 5.17	+ 0.478	+ 0.898	+ 1.02	+ 0.473	+ 0.901	+ 1.02
α Cygni	20 37 0.07	. .	. .	. .	− 0.140	+ 1.40	+ 1.41
μ Aquarii	20 45 38.47	. .	. .	. .	+ 0.747	+ 0.674	+ 1.01
ν Cygni	20 52 19.70	. .	. .	. .	− 0.046	+ 1.32	+ 1.32
ι Pegasi	21 16 4.52	+ 0.357	+ 0.997	+ 1.06			
β Aquarii	21 24 42.90	+ 0.712	+ 0.710	+ 1.01			

Washington Observations.—The observations for time made at Washington are given in Table II. The first and second columns do not require any explanation. The column "*Observer*" contains the initials of the person who made the observations, as follows:

Ha. . . . . Professor William Harkness.
E. . . . . Professor John R. Eastman.
F. . . . . Assistant Observer Edgar Frisby.
S. . . . . Assistant Observer Ormond Stone.

The column "*No. of Wires*" gives the number of wires over which the transit of the star was observed. All time-stars were observed by the chronographic method, and, as a rule, over nine wires; but azimuth-stars were observed by eye and ear, and generally over only five wires. The column "*Time of Transit over Mean of Wires*" contains the time of transit over an imaginary wire situated at the mean of the standard set of nine wires. For stars observed over all the wires of that set the mean of the observed times of transit is of course the time of transit over the mean wire, but for other stars the time of transit over the mean wire has been deduced from the observed times of transit by the application of the necessary corrections. The columns "*Cc*," "*Bb*," and "*Aa*" contain the corrections for collimation, level, and azimuth. The numbers in the column "*Correction for Instrument*" are the sums of the quantities in the three preceding columns. The column "*Corr. Transit*" contains the clock-time of transit over the meridian, obtained by adding together the quantities in the columns "*Time of Transit over Mean of Wires*" and "*Correction for Instrument.*" The column "*Adopted Right Ascension*" contains the adopted apparent right ascensions of the stars observed. The column "*Observed Clock Corr.*" contains the difference between the "*Corr. Transit*" and the

"*Adopted Right Ascension.*" The column v contains the difference between the observed and adopted clock corrections; or, in other words, the error of observation.

The values of the constants employed during each night are as follows:

Date.	c	b	a
	s.	s.	s.
April 12	− 0.02	− 0.15	+ 0.02
23	.01	.14	− 0.15
26	.02	.08	.07
30	− 0.02	− 0.11	− 0.16

The constant c was obtained from observations on a pair of opposing collimators. b was obtained from observations of the spirit-level, two readings being made with it in a direct, and two with it in a reversed position. a was computed from the observed transits of Polaris, using for that purpose a closely approximate value of the clock correction. Full details as to the methods of determining these constants are given on pages xxvi–xxviii of the Washington Observations for 1870.

Table II.—*Transits of Stars observed at Washington to determine the Corrections to the Kessels Sidereal Clock.*

Date.	Star.	Observer.	No. of Wires.	Time of Transit over Mean of Wires.	*Cc*	*Bb*	*Aa*	Correction for Instrument.	Corr. Transit.	Adopted Right Ascension.	Observed Clock Corr.	*v*
1870.				h. m. s.	s.	s.	s.	s.	s.	s.	s.	
April 12	τ Leonis . .	Ha.	9	11 21 18.11	−0.12	−0.12	+0.01	−0.13	17.98	16.00	−1.98	+.02
	υ Leonis . .	Ha.	9	30 20.54	.02	.12	.01	.13	20.41	18.51	1.90	−.06
	β Leonis . .	Ha.	9	42 28.72	.02	.14	.01	.15	28.57	26.57	2.00	+.04
	o Virginis . .	Ha.	9	11 58 38.29	.02	.13	.01	.14	38.15	36.20	1.95	−.02
	η Virginis . .	Ha.	9	12 13 18.50	.02	.12	.01	.13	18.37	16.37	2.00	+.03
	θ Virginis . .	Ha.	9	13 3 16.48	− .02	−0.11	.01	−0.12	16.36	14.39	1.97	−.01
	Polaris, S. P. .	Ha.	5	13 10 29.50	+0.83	+3.78	+0.66	+5.27	34.77	32.75	−2.02	
April 23	υ Leonis . .	F.	9	11 30 24.28	−0.01	−0.11	−0.09	−0.21	24.07	18.45	−5.62	−.01
	β Leonis . .	F.	9	42 32.40	.01	.13	.06	.20	32.20	26.51	.69	+.05
	o Virginis . .	F.	9	11 58 41.96	.01	.12	.07	.20	41.76	36.15	.61	−.04
	η Virginis . .	F.	9	12 13 22.21	.01	.11	− .09	.21	22.00	16.35	.65	−.01
	12 Canum Venat.	F.	9	12 49 3.76	− .01	−0.18	0.00	.19	3.57	57.88	.69	+.01
	Polaris, S. P. .	F.	5	13 10 42.10	+ .41	+3.53	−4.93	.99	41.11	35.27	.84	
	μ^1 Bootis. . .	F.	9	15 19 42.16	− .01	−0.18	0.00	.19	41.97	36.24	.73	−.03
	α Cor. Borealis.	F.	9	15 29 18.31	−0.01	−0.15	−0.03	−0.19	18.12	12.33	−5.79	+.03
April 26	τ Leonis . .	F.	5	11 21 23.19	−0.02	−0.07	−0.04	−0.13	23.06	15.90	−7.16	−.03
	υ Leonis . .	F.	9	30 25.83	.02	.06	.04	.12	25.71	18.43	.28	+.08
	o Virginis . .	F.	9	11 58 43.43	.02	.07	− .04	.13	43.30	36.13	.17	−.05
	12 Canum Venat.	F.	9	12 50 5.20	.03	.10	.00	.13	5.07	57.87	.20	−.05
	θ Virginis . .	Ha.	9	13 3 21.70	− .02	−0.06	−0.05	− .13	21.57	14.41	.16	+.06
	Polaris, S. P. .	F.	5	10 42.68	+ .83	+2.02	−2.30	+ .55	43.23	35.95	.28	
	ζ Virginis . .	F.	9	28 12.88	− .02	−0.06	0.04	− .12	12.76	5.42	.34	+.06
	η Bootis. . .	Ha.	9	13 48 38.14	.02	.08	.03	.13	38.01	30.91	.10	−.03
	α Serpentis . .	F.	9	15 38 0.63	.02	.07	.04	.13	0.50	53.15	.35	−.02
	ε Serpentis . .	Ha.	9	15 44 28.79	−0.02	−0.07	−0.04	−0.13	28.66	21.44	−7.22	+.01
April 30	o Virginis . .	Ha.	9	11 58 45.19	−0.02	−0.10	−0.08	−0.20	44.99	36.11	−8.88	−.01
	η Virginis . .	Ha.	9	12 13 25.43	.02	.09	.10	.21	25.22	16.32	.90	+.01
	β Corvi . . .	Ha.	9	12 27 44.11	− .02	−0.06	0.15	0.23	43.88	34.93	.95	+.05
	Polaris, S. P. .	Ha.	5	13 10 48.10	+ .83	+2.77	5.26	1.66	46.44	37.53	.91	
	α Virginis . .	Ha.	9	18 31.12	− .02	−0.07	0.12	0.21	30.91	22.06	.85	−.06
	ζ Virginis . .	Ha.	9	28 14.54	.02	.09	.10	.21	14.33	5.43	.90	−.01
	η Bootis. . .	Ha.	9	13 48 40.00	.02	.11	.06	.19	39.81	30.93	.88	−.03
	α' Bootis. . .	Ha.	9	14 9 54.37	−0.02	−0.11	−0.06	−0.19	54.18	45.20	−8.98	+.06

Each of the quantities in the column "*Observed Clock Corr.*" is equal to

$$\alpha - [T' + Aa + Bb + Cc]$$

which, for brevity, we will represent by n. Then each star observed furnishes an equation of condition of the form

$$0 = -n + \Delta T_0 + R\,(T' - T_0) + v$$

and from all the equations thus obtained on any given night the values of ΔT_0 and R for that night have been found by the method of least squares. Assuming $T_0 = 11^{h}\,0^{m}$ by the face of the Kessels clock, the equations of condition, normal equations, and resulting values of ΔT_0 and R, for each night, are as follows:

WASHINGTON, APRIL 12, 1870.

Equations of Condition.

$$\begin{aligned}
0 &= +\overset{s.}{1.98} + \Delta T_0 + \overset{h.}{0.35}\,R \\
0 &= +1.90 + \Delta T_0 + 0.50\,R \\
0 &= +2.00 + \Delta T_0 + 0.71\,R \\
0 &= +1.95 + \Delta T_0 + 0.98\,R \\
0 &= +2.00 + \Delta T_0 + 1.22\,R \\
0 &= +1.97 + \Delta T_0 + 2.05\,R
\end{aligned}$$

Normal Equations.

$$\begin{aligned}
0 &= +\overset{s.}{11.80} + 6.00\,\Delta T_0 + 5.81\,R \\
0 &= +11.45 + 5.81\,\Delta T_0 + 7.53\,R
\end{aligned}$$

Hence

$$\begin{aligned}
\Delta T_0 &= -\overset{s.}{1.954} \pm \overset{s.}{0.010} \\
R &= -0.0133
\end{aligned}$$

WASHINGTON, APRIL 23, 1870.

Equations of Condition.

$$\begin{aligned}
0 &= +\overset{s.}{5.62} + \Delta T_0'' + \overset{h.}{0.51}\,R \\
0 &= +5.69 + \Delta T_0'' + 0.71\,R \\
0 &= +5.61 + \Delta T_0'' + 0.98\,R \\
0 &= +5.65 + \Delta T_0'' + 1.22\,R \\
0 &= +5.69 + \Delta T_0'' + 1.82\,R \\
0 &= +5.73 + \Delta T_0'' + 4.33\,R \\
0 &= +5.79 + \Delta T_0'' + 4.49\,R
\end{aligned}$$

Normal Equations.

$$\begin{aligned}
0 &= +\overset{s.}{39.78} + 7.00\,\Delta T_0'' + 14.06\,R \\
0 &= +80.46 + 14.06\,\Delta T_0'' + 45.43\,R
\end{aligned}$$

Hence

$$\begin{aligned}
\Delta T_0'' &= -\overset{s.}{5.615} \pm \overset{s.}{0.008} \\
R &= -0.0335
\end{aligned}$$

WASHINGTON, APRIL 26, 1870.

Equations of Condition.

$$\begin{aligned}
0 &= +\overset{s.}{7.16} + \Delta T_0'' + \overset{h.}{0.36}\,R \\
0 &= +7.28 + \Delta T_0'' + 0.51\,R \\
0 &= +7.17 + \Delta T_0'' + 0.98\,R \\
0 &= +7.20 + \Delta T_0'' + 1.83\,R \\
0 &= +7.16 + \Delta T_0 + 2.06\,R
\end{aligned}$$

$$\begin{aligned}
0 &= +\overset{s.}{7.34} + \Delta T_0'' + \overset{h.}{2.47}\,R \\
0 &= +7.10 + \Delta T_0 + 2.81\,R \\
0 &= +7.35 + \Delta T_0'' + 4.63\,R \\
0 &= +7.22 + \Delta T_0 + 4.74\,R
\end{aligned}$$

Normal Equations.

$$
\begin{aligned}
0 &= + 43^{s}.50 + 6.00\,\varDelta T_0'' + 0.00\,\varDelta T_0 + 10.78\,R \\
0 &= + 21.48 + 0.00\,\varDelta T_0'' + 3.00\,\varDelta T_0 + 9.61\,R \\
0 &= + 147.58 + 10.78\,\varDelta T_0'' + 9.61\,\varDelta T_0 + 66.84\,R
\end{aligned}
$$

Hence

$$
\begin{aligned}
\varDelta T_0'' &= - 7^{s}.176 \pm 0^{s}.016 \\
\varDelta T_0 &= - 7.013 \pm 0.019 \\
R &= - 0.0423
\end{aligned}
$$

WASHINGTON, APRIL 30, 1870.

Equations of Condition.

$$
\begin{aligned}
0 &= + 8^{s}.88 + \varDelta T_0 + 0^{h}.98\,R \\
0 &= + 8.90 + \varDelta T_0 + 1.22\,R \\
0 &= + 8.95 + \varDelta T_0 + 1.46\,R \\
0 &= + 8.85 + \varDelta T_0 + 2.31\,R \\
0 &= + 8.90 + \varDelta T_0 + 2.47\,R \\
0 &= + 8.88 + \varDelta T_0 + 2.81\,R \\
0 &= + 8.98 + \varDelta T_0 + 3.16\,R
\end{aligned}
$$

Normal Equations.

$$
\begin{aligned}
0 &= + 62^{s}.34 + 7.00\,\varDelta T_0 + 14.41\,R \\
0 &= + 128.38 + 14.41\,\varDelta T_0 + 33.90\,R
\end{aligned}
$$

Hence

$$
\begin{aligned}
\varDelta T_0 &= - 8^{s}.880 \pm 0^{s}.011 \\
R &= - 0.0121
\end{aligned}
$$

As will be shown farther on, Mr. Frisby observes the transit of an equatorial star $0^{s}.121$ later than I, and therefore we have

$$\varDelta T_0 = \varDelta T_0'' + 0^{s}.121$$

Hence, on April 23,

$$\varDelta T_0 = - 5^{s}.615 + 0^{s}.121 = - 5^{s}.494 \pm 0^{s}.008$$

On April 26, Mr. Frisby's observations give

$$\varDelta T_0 = - 7^{s}.176 + 0^{s}.121 = - 7^{s}.055 \pm 0^{s}.016$$

and my own give

$$\varDelta T_0 = - 7^{s}.013 \pm 0^{s}.019$$

Taking the mean, we find

$$\varDelta T_0 = - 7^{s}.034 \pm 0^{s}.014$$

Collecting our results, we have the expressions given in Table III for the corrections which must be applied to the time indicated by the face of the Kessels clock, in order to reduce it to sidereal time determined by myself at the meridian of the transit circle. T' is the time indicated by the clock at the instant for which the correction is required, and the quantities after the sign $\pm$ are approximately the probable errors.

TABLE III.—*Corrections to the Kessels Clock.*

Date.		Correction.
		s. s. h. s.
April	12	$- 1.954 - 0.0133\,(T' - 11.00) \pm 0.010$
	23	$- 5.494 - 0.0335\,(T' - 11.00) \pm 0.008$
	26	$- 7.034 - 0.0423\,(T' - 11.00) \pm 0.014$
	30	$- 8.880 - 0.0121\,(T' - 11.00) \pm 0.011$

St. Louis Observations.—The observations for time at St. Louis were made by Professor William Eimbeck, and are given in Table IV. The first column contains the date. The column "*Lamp*" gives the position of the axis of the transit instrument; E. signifying that the lamp was to the east, W. that it was to the west. The column "*No. of Wires*" gives the number of wires over which the transit of the star was observed. The field of view of the instrument contained nine vertical wires, separated by intervals of about 15 seconds of time, but as a rule only the middle seven wires were used, and all observations were made by the eye and ear method. The column "*Star*" does not require any explanation. The column "*Time of Transit over Mean of Wires*" contains the time of transit over an imaginary wire situated at the mean of the standard set of seven wires. For stars not observed over all the wires of that set the time of transit over the mean wire has been deduced from the observed times of transit by the application of the necessary corrections. The column "*b*" contains the observed values of the level constant, each of them being derived from two readings of the spirit-level, one made with it in the direct, the other with it in the reversed position. The level is of the striding form, and each division of its scale is equal to $0^s.09$. The columns "*Bb*" and "*Cc*" contain the corrections for level and collimation. The column "*r*" contains the correction for rate of the chronometer. The column "*Corr. Transit*" contains the sum of the quantities in the columns "*Time of Transit over Mean of Wires*," "*Bb*," "*Cc*" and "*r*." The column "*Adopt'd Right Ascension*" contains the adopted apparent right ascensions of the stars observed. The column "*Obs'd Chron Correction*" contains the difference between the "*Corr. Transit*" and the "*Adopt'd Right Ascension.*" The column "*v*" contains the difference between the observed and adopted chronometer correction; or, in other words, the error of observation.

Throughout the whole series of time determinations the adopted value of B is $-0^s.086$; and the adopted value of c is $+0^s.24$ for lamp west. The latter constant was obtained from transits of circumpolar stars, each observation being made over one-half the wires with the lamp west, and over the other half with lamp east.

TABLE IV.—*Transits of Stars observed at St. Louis to determine the Corrections to the Sidereal Chronometer Kessels and Dent No.* 1287.

Date.	Lamp.	No. of Wires.	Star.	Time of Transit over Mean of Wires.	b	Bb	Cc	r	Corr. Transit.	Adopt'd Right Ascension.	Obs'd Chron. Correction.	v
1870.				h. m. s.	s.	s.	s.	s.	m. s.	m. s.	m. s.	s.
April 12	E.	7	ϵ Hydræ . .	8 33 14.99	−0.09	−0.08	−0.24	−0.05	33 14.62	39 53.60	+6 38.98	−.02
	E.	7	ι Ursæ Maj. .	43 40.78	.13	.19	.36	.07	43 40.16	50 17.91	37.75	+.04
	E.	7	κ Cancri . .	8 54 4.01	.19	0.17	0.24	.08	54 3.52	0 42.51	38.99	−.13
	E.	4	1 Draconis .	9 11 54.42	. .	1.09	1.70	.11	11 51.52	18 22.27	30.75	+.06
	E.	7	α Hydræ . .	14 33.60	. .	−0.15	−0.24	.12	14 33.09	21 12.38	39.29	−.02
	E.	5	β Cephei, S.P.	20 14.21	.23	+ .22	+ .70	.12	20 15.01	26 57.21	42.20	+.14
	W.	7	ϵ Leonis . .	31 50.04	.25	+ .27	.26	.13	31 49.90	38 28.54	38.64	−.05
	W.	7	μ Leonis . .	38 43.89	.17	.19	.27	.15	38 43.82	45 22.41	38.59	−.08
	W.	4	α Leonis . .	9 54 48.57	.09	.09	.24	.17	54 48.55	1 27.37	38.82	+.02
	W.	7	β Libræ . .	15 3 23.16	.20	.14	.24	.01	3 23.25	10 1.90	38.65	+.02
	W.	7	μ^1 Bootis . .	11 58.53	. .	.24	.30	.03	11 58.56	19 36.07	37.51	+.09
	W.	3	γ^2 Ursæ Min. .	14 25.43	.19	.52	.79	.03	14 25.67	21 0.50	34.83	+.10
	W.	5	α Coronæ Bor.	22 34.24	.17	.20	.27	.04	22 34.27	29 12.16	37.89	+.02
	W.	7	α Serpentis .	31 14.53	. .	.15	+ .24	.05	31 14.57	37 52.94	38.37	−.02
	E.	7	ϵ Serpentis .	37 43.21	. .	.15	−0.24	.06	37 42.76	44 21.21	38.45	−.06
	E.	5	ζ Ursæ Min. .	42 19.07	. .	.68	1.17	.06	42 17.16	48 50.29	33.13	−.09
	E.	6	β^1 Scorpii . .	15 51 15.33	−0.18	−0.10	−0.25	−0.07	51 14.91	57 53.86	+6 38.95	−.06
April 23	E.	5	1 Draconis .	9 11 53.62	+0.10	+0.52	−1.70	−0.03	11 52.41	18 20.82	+6 28.41	+.29
	E.	7	α Hydræ . .	14 46.81	. .	+ .05	−0.24	.03	14 46.59	21 12.22	25.63	−.06
	E.	6	β Cephei, S.P.	20 33.02	+ .04	− .04	+ .70	.03	20 33.65	26 57.90	24.25	+.28
	E.	7	ϵ Leonis . .	32 2.90	. .	.00	− .26	.05	32 2.59	38 28.38	25.79	+.05
	E.	7	μ Leonis . .	38 56.97	− .11	− .12	.27	.06	38 56.52	45 22.25	25.73	+.13
	E.	7	α Leonis . .	9 55 1.96	.16	.14	0.24	.08	55 1.50	1 27.23	25.73	.00
	E.	7	9 Draconis .	10 17 34.12	.18	.60	−1.02	.12	17 32.38	24 0.14	27.76	−.31
	W.	7	l Leonis . .	36 0.31	.25	.22	+0.24	.14	36 0.19	42 25.99	25.80	−.08
	W.	7	α Ursæ Maj. .	10 49 15.53	.20	.40	.52	.16	49 15.49	55 42.23	26.74	−.26
	W.	7	δ Leonis . .	11 0 46.54	. .	.18	.25	.17	0 46.44	7 12.25	25.81	.00
	W.	6	δ Crateris .	6 26.13	.17	.11	.24	.18	6 26.08	12 51.45	25.37	+.16
	W.	7	λ Draconis .	17 14.56	.15	.38	.70	.20	17 14.68	23 41.50	26.82	+.05
	W.	7	β Leonis . .	36 0.89	. .	.14	.24	.22	36 0.77	42 26.51	25.74	+.02
	W.	7	γ Ursæ Maj. .	40 33.90	.15	.24	0.41	.23	40 33.84	47 0.29	26.45	−.20
	W.	4	4 Draconis .	11 59 40.77	−0.15	−0.57	+1.19	−0.26	59 41.13	6 8.85	+6 27.72	−.06
April 26	W.	4	1 Draconis .	9 11 56.91	0.00	0.00	+1.70	−0.02	11 58.59	18 20.42	+6 21.83	+.65
	W.	7	α Hydræ . .	14 52.59	. .	.00	+0.24	.03	14 52.80	21 12.18	19.38	.00
	W.	7	β Cephei, S.P.	20 40.66	− .07	+ .07	− .70	.03	20 40.00	26 58.12	18.12	+.13
	W.	7	ϵ Leonis . .	32 8.46	.05	− .05	+ .26	.05	32 8.62	38 28.33	19.71	−.07
	W.	7	μ Leonis . .	39 2.47	.08	.09	.27	.06	39 2.59	45 22.20	19.61	+.05
	W.	7	α Leonis . .	9 55 7.49	.02	.02	.24	.08	55 7.63	1 27.17	19.54	.00
	W.	6	γ^1 Leonis . .	10 6 28.93	− .01	− .01	0.25	.10	6 29.07	12 48.56	19.49	+.11
	W.	5	9 Draconis .	17 36.82	+ .04	+ .13	+1.02	.12	17 37.85	23 59.94	22.09	−.86
	E.	4	α Ursæ Maj. .	10 49 22.66	0.00	0.00	−0.52	−0.16	49 21.98	55 42.15	+6 20.17	+.20

TABLE IV.—*Transits of Stars observed at St. Louis, &c.*—Continued.

Date.	Lamp.	No. of Wires.	Star.	Time of Transit over Mean of Wires.	b	Bb	Cc	r	Corr. Transit.	Adopt'd Right Ascension.	Obs'd Chron. Correction.	v
1870.				h. m. s.	s.	s.	s.	s.	m. s.	m. s.	m. s.	s.
April 26	E.	5	δ Leonis . .	11 0 53.07	−0.05	−0.05	−0.25	−0.17	0 52.60	7 12.23	+6 19.63	−.02
	E.	5	λ Draconis .	17 21.79	.06	.15	.70	.20	17 20.74	23 41.38	20.64	.00
	E.	7	β Leonis . .	36 7.57	. .	.09	.25	.22	36 7.01	42 26.49	19.48	+.08
	E.	7	γ Ursæ Maj. .	40 40.93	.09	.15	.41	.23	40 40.14	47 0.25	20.11	−.07
	E.	7	o Virginis .	52 17.26	.12	.10	0.24	.24	52 16.68	58 36.13	19.45	+.06
	E.	6	4 Draconis .	11 59 48.90	.12	.46	1.19	− .26	59 46.99	6 8.69	21.70	−.16
	E.	7	α² Libræ . .	14 37 24.30	. .	.06	0.24	+ .02	37 24.02	43 42.74	18.72	+.02
	E.	7	β Ursæ Min. .	44 51.66	. .	.31	.91	+ .01	44 50.45	51 10.95	20.50	−.01
	E.	5	β Bootis . .	14 50 45.57	.10	.13	.31	.00	50 45.13	57 4.39	19.26	+.01
	E.	7	β Libræ . .	15 3 43.73	.15	.10	.24	− .01	3 43.38	10 2.11	18.73	+.07
	E.	7	μ¹ Bootis . .	13 17.57	.20	.25	.30	.02	13 17.00	19 36.27	19.27	−.04
	E.	7	α Coronæ Bor.	22 53.82	.21	.23	.27	.04	22 53.28	29 12.38	19.10	.00
	E.	7	α Serpentis .	31 34.80	.19	.17	−0.24	.05	31 34.34	37 53.17	18.83	+.09
	W.	5	ζ Ursæ Min. .	42 29.61	.21	.79	+1.17	.06	42 29.93	48 50.97	21.04	+.04
	W.	7	ε Coronæ Bor.	45 54.58	. .	.23	0.27	.07	45 54.55	52 13.70	19.15	−.03
	W.	4	β¹ Scorpii . .	51 35.39	.17	.10	.25	.08	51 35.46	57 54.15	18.69	+.01
	W.	5	Groom. 2320	15 59 41.16	. .	.28	.65	.09	59 41.44	6 1.40	19.96	+.06
	W.	7	τ Herculis .	16 9 32.17	.07	.10	.35	.11	9 32.31	15 51.70	19.39	−.02
	W.	5	η Draconis .	15 56.34	.06	.12	.51	.12	15 56.61	22 16.44	19.83	−.10
	W.	7	ζ Ophiuchi .	23 42.36	. .	.03	.24	.12	23 42.45	30 1.27	18.82	−.03
	W.	6	η Herculis .	16 36 8.33	−0.04	−0.05	+0.31	−0.13	36 8.64	38 27.76	+6 19.30	.00
April 30	W.	2	1 Draconis .	9 12 6.15	. .	0.00	+1.70	−0.02	12 7.83	18 19.87	+6 12.04	−.62
	W.	7	α Hydræ . .	14 59.36	. .	.00	+0.24	.03	14 59.57	21 12.12	12.55	+.07
	W.	5	β Cephei, S. P.	20 46.08	− .02	+ .02	− .70	.03	20 45.37	26 58.36	12.99	+.06
	W.	7	ε Leonis . .	32 15.66	.08	− .09	+ .26	.05	32 15.78	38 28.27	12.49	+.03
	W.	7	μ Leonis . .	39 9.56	.08	− .09	+ .27	.06	39 9.68	45 22.14	12.46	+.05
	W.	3	79 Drac., S. P.	45 2.00	. .	+ .11	− .83	.07	45 1.21	51 14.50	13.29	−.15
	W.	7	α Leonis . .	9 55 14.47	.11	− .10	+ .24	.08	55 14.53	1 27.14	12.61	−.05
	W.	4	32 Ursæ Maj. .	10 2 21.81	.08	.17	0.59	.09	2 22.14	8 34.35	12.21	+.01
	W.	7	9 Draconis .	17 46.74	− .02	− .07	1.02	.12	17 47.57	23 59.64	12.07	−.16
	W.	7	l Leonis . .	36 13.16	. .	.00	+0.24	.14	36 13.26	42 25.91	12.65	−.09
	W.	4	ι Cephei, S. P.	38 49.86	.00	.00	− .58	.15	38 49.13	45 2.49	13.36	−.40
	W.	7	α Ursæ Maj. .	10 49 29.29	+ .04	+ .08	+ .52	.16	49 29.73	55 42.01	12.28	−.10
	E.	7	δ Leonis . .	11 0 59.94	+ .02	+ .02	− .25	.17	0 59.54	7 12.18	12.64	−.11
	E.	5	δ Crateris .	6 39.22	. .	− .03	.24	.18	6 38.77	12 51.37	12.60	+.04
	E.	6	τ Leonis . .	15 3.83	. .	.06	.24	.19	15 3.34	21 15.87	12.53	+.05
	E.	4	λ Draconis .	17 30.35	− .11	.28	.70	.20	17 29.17	23 41.22	12.05	+.08
	E.	7	β Leonis . .	36 14.61	. .	.18	.25	.22	36 13.96	42 26.46	12.50	+.05
	E.	7	γ Ursæ Maj. .	40 48.93	.23	.38	.41	.23	40 47.91	47 0.17	12.26	+.10
	E.	7	o Virginis .	52 24.36	.29	0.26	0.24	.25	52 23.61	58 36.11	12.50	+.07
	E.	5	4 Draconis .	11 59 59.70	. .	1.10	1.19	.26	59 57.15	6 8.45	11.30	+.48
	E.	7	η Virginis .	12 7 4.55	−0.30	0.23	0.24	.26	7 3.82	13 16.31	12.49	+.10
	E.	6	κ Draconis .	12 21 47.81	. .	−0.76	−0.72	−0.29	21 64.04	27 57.78	+6 11.74	+.14

3—W S

Each of the quantities in the column "*Obs'd Chron. Correction*" is equal to

$$\alpha - [T' + R\,(T' - T_0) + Bb + Cc]$$

which, for brevity, may be represented by n. Then we have

$$0 = -n + \varDelta T_0 + Aa + v$$

But as throughout this series of observations n is very large, we write

$$\varDelta T_0 = \theta + \delta\theta$$

and the equation just given becomes

$$0 = -n + \theta + \delta\theta + Aa + v$$

in which the absolute term, $-n + \theta$, may be made sufficiently small by choosing a suitable value of θ. Each star observed furnishes an equation of condition of this form, and from all the equations so obtained on any given night the values of $\delta\theta$ and a for that night have been found by the method of least squares.

The adopted values of T_0 and θ for each night, together with the equations of condition, normal equations, and resulting values of $\delta\theta$ and a, are as follows:

St. Louis, April 12, 1870.

Equations of Condition; 1st Group, 8^h to 10^h.

$T_0 = 7^h\ 53^m\ 22^s$	$\theta = +\ 6^m\ 30^s.000$
$0 = -\ 8^s.98 + \delta\theta + 0.529\ a$	$0 = -\ 12^s.20 + \delta\theta + 2.760\ a$
$0 = -\ 7.75 + \delta\theta - 0.274\ a$	$0 = -\ 8.64 + \delta\theta + 0.267\ a$
$0 = -\ 8.99 + \delta\theta + 0.462\ a$	$0 = -\ 8.59 + \delta\theta + 0.232\ a$
$0 = -\ 0.75 + \delta\theta - 4.850\ a$	$0 = -\ 8.82 + \delta\theta + 0.449\ a$
$0 = -\ 9.29 + \delta\theta + 0.734\ a$	

Normal Equations.

$$0 = -\ 74^s.01 + 9.00\ \delta\theta + 0.34\ a$$
$$0 = -\ 52.10 + 0.34\ \delta\theta + 32.56\ a$$

Hence

$$\delta\theta = +0^m\ 8^s.163$$
$$a = +0\ \ 1.515$$
$$\varDelta T_0 = +6\ \ 38.163 \pm 0^s.019$$

Equations of Condition; 2d Group, 15^h to 16^h.

$T_0 = 14^h\ 53^m\ 22^s$	$\theta = +\ 6^m\ 30^s.000$
$0 = -\ 8^s.65 + \delta\theta + 0.746\ a$	$0 = -\ 8^s.37 + \delta\theta + 0.530\ a$
$0 = -\ 7.51 + \delta\theta + 0.017\ a$	$0 = -\ 8.45 + \delta\theta + 0.557\ a$
$0 = -\ 4.83 + \delta\theta - 1.820\ a$	$0 = -\ 3.13 + \delta\theta - 3.120\ a$
$0 = -\ 7.89 + \delta\theta + 0.223\ a$	$0 = -\ 8.95 + \delta\theta + 0.899\ a$

Normal Equations.

$$0 = -\overset{\text{s.}}{57.78} + 8.00\,\delta\theta - 1.97\,a$$
$$0 = -6.97 - 1.97\,\delta\theta + 15.05\,a$$

Hence

$$\delta\theta = +0^{\text{m.}}\ 7^{\text{s.}}.582$$
$$a = +0\ \ 1.455$$
$$\varDelta T_0 = +6\ \ 37.582 \pm 0^{\text{s}}.018$$

ST. LOUIS, APRIL 23, 1870.

Equations of Condition.

$T_0 = 8^{\text{h}}\ 53^{\text{m}}\ 34^{\text{s}}$	$\theta = +6^{\text{m}}\ 20^{\text{s}}.000$
$0 = -\overset{\text{s.}}{8.41} + \delta\theta - 4.850\,a$	$0 = -\overset{\text{s.}}{6.74} + \delta\theta - 0.875\,a$
$0 = -5.63 + \delta\theta + 0.734\,a$	$0 = -5.81 + \delta\theta + 0.320\,a$
$0 = -4.25 + \delta\theta + 2.760\,a$	$0 = -5.37 + \delta\theta + 0.820\,a$
$0 = -5.79 + \delta\theta + 0.267\,a$	$0 = -6.82 + \delta\theta - 1.530\,a$
$0 = -5.73 + \delta\theta + 0.232\,a$	$0 = -5.74 + \delta\theta + 0.411\,a$
$0 = -5.73 + \delta\theta + 0.449\,a$	$0 = -6.45 + \delta\theta - 0.468\,a$
$0 = -7.76 + \delta\theta - 2.600\,a$	$0 = -7.72 + \delta\theta - 3.160\,a$
$0 = -5.80 + \delta\theta + 0.469\,a$	

Normal Equations.

$$0 = -\overset{\text{s.}}{93.75} + 15.00\,\delta\theta - 7.02\,a$$
$$0 = +72.07 - 7.02\,\delta\theta + 53.24\,a$$

Hence

$$\delta\theta = +0^{\text{m.}}\ 5^{\text{s.}}.988$$
$$a = -0\ \ 0.564$$
$$\varDelta T_0 = +6\ \ 25.988 \pm 0^{\text{s}}.033$$

ST. LOUIS, APRIL 26, 1870.

Equations of Condition; 1st Group, 9^{h} *to* 12^{h}.

$T_0 = 8^{\text{h}}\ 53^{\text{m}}\ 40^{\text{s}}$	$\theta = +6^{\text{m}}\ 18^{\text{s}}.000$
$0 = -\overset{\text{s.}}{3.83} + \delta\theta - 4.850\,a$	$0 = -\overset{\text{s.}}{2.17} + \delta\theta - 0.875\,a$
$0 = -1.38 + \delta\theta + 0.734\,a$	$0 = -1.63 + \delta\theta + 0.320\,a$
$0 = -0.12 + \delta\theta + 2.760\,a$	$0 = -2.64 + \delta\theta - 1.530\,a$
$0 = -1.71 + \delta\theta + 0.267\,a$	$0 = -1.48 + \delta\theta + 0.411\,a$
$0 = -1.61 + \delta\theta + 0.232\,a$	$0 = -2.11 + \delta\theta - 0.468\,a$
$0 = -1.54 + \delta\theta + 0.449\,a$	$0 = -1.45 + \delta\theta + 0.493\,a$
$0 = -1.49 + \delta\theta + 0.331\,a$	$0 = -3.70 + \delta\theta - 3.160\,a$
$0 = -4.09 + \delta\theta - 2.600\,a$	

Normal Equations.

$$0 = -\overset{\text{s.}}{30.95} + 15.00\,\delta\theta - 7.49\,a$$
$$0 = +42.64 - 7.49\,\delta\theta + 52.70\,a$$

Hence

$$\delta\theta = +\overset{\text{m.}}{0}\ \overset{\text{s.}}{1.786}$$
$$a = -0\ \ 0.555$$
$$\Delta T_0 = +6\ \ 19.786 \pm 0^{\text{s}}.057$$

Equations of Condition; 2d Group, 14^h to 17^h.

$T_0 = 14^{\text{h}}\,53^{\text{m}}\,41^{\text{s}}$ | $\theta = +6^{\text{m}}\,18^{\text{s}}.000$

$$0 = -\overset{\text{s.}}{0.72} + \delta\theta + 0.838\,a$$
$$0 = -2.50 + \delta\theta - 2.130\,a$$
$$0 = -1.26 + \delta\theta - 0.052\,a$$
$$0 = -0.73 + \delta\theta + 0.746\,a$$
$$0 = -1.27 + \delta\theta + 0.017\,a$$
$$0 = -1.10 + \delta\theta + 0.223\,a$$
$$0 = -0.83 + \delta\theta + 0.530\,a$$
$$0 = -3.04 + \delta\theta - 3.120\,a$$

$$0 = -\overset{\text{s.}}{1.15} + \delta\theta + 0.222\,a$$
$$0 = -0.69 + \delta\theta + 0.899\,a$$
$$0 = -1.96 + \delta\theta - 1.330\,a$$
$$0 = -1.39 + \delta\theta - 0.189\,a$$
$$0 = -1.83 + \delta\theta - 0.836\,a$$
$$0 = -0.82 + \delta\theta + 0.764\,a$$
$$0 = -1.30 + \delta\theta - 0.011\,a$$

Normal Equations.

$$0 = -\overset{\text{s.}}{20.59} + 15.00\,\delta\theta - 3.43\,a$$
$$0 = +15.92 - 3.43\,\delta\theta + 19.81\,a$$

Hence

$$\delta\theta = +\overset{\text{m.}}{0}\ \overset{\text{s.}}{1.237}$$
$$a = -0\ \ 0.590$$
$$\Delta T_0 = +6\ \ 19.237 \pm 0^{\text{s}}.009$$

St. Louis, April 30, 1870.

Equations of Condition.

$T_0 = 8^{\text{h}}\,53^{\text{m}}\,48^{\text{s}}$ | $\theta = +6^{\text{m}}\,12^{\text{s}}.000$

$$0 = -\overset{\text{s.}}{0.04} + \delta\theta - 4.850\,a$$
$$0 = -0.55 + \delta\theta + 0.734\,a$$
$$0 = -0.99 + \delta\theta + 2.760\,a$$
$$0 = -0.49 + \delta\theta + 0.267\,a$$
$$0 = -0.46 + \delta\theta + 0.232\,a$$
$$0 = -1.29 + \delta\theta + 3.190\,a$$
$$0 = -0.61 + \delta\theta + 0.449\,a$$
$$0 = -0.21 + \delta\theta - 1.110\,a$$
$$0 = -0.07 + \delta\theta - 2.600\,a$$
$$0 = -0.65 + \delta\theta + 0.469\,a$$
$$0 = -1.36 + \delta\theta + 2.340\,a$$

$$0 = -\overset{\text{s.}}{0.28} + \delta\theta - 0.875\,a$$
$$0 = -0.64 + \delta\theta + 0.320\,a$$
$$0 = -0.60 + \delta\theta + 0.820\,a$$
$$0 = -0.53 + \delta\theta + 0.574\,a$$
$$0 = -0.05 + \delta\theta - 1.530\,a$$
$$0 = -0.50 + \delta\theta + 0.411\,a$$
$$0 = -0.26 + \delta\theta - 0.468\,a$$
$$0 = -0.50 + \delta\theta + 0.493\,a$$
$$0 = +0.70 + \delta\theta - 3.160\,a$$
$$0 = -0.49 + \delta\theta + 0.622\,a$$
$$0 = +0.26 + \delta\theta - 1.580\,a$$

Normal Equations.

$$0 = -\ \overset{\text{s.}}{9.61} + 22.00\,\delta\theta - 2.49\,a$$
$$0 = -\ 14.56 - 2.49\,\delta\theta + 73.59\,a$$

Hence

$$\delta\theta = +\overset{\text{m.}}{0}\ \overset{\text{s.}}{0.461}$$
$$a = +0\ 0.214$$
$$\varDelta T_0 = +6\ 12.461 \pm 0^{\text{s}}.031$$

These values of $\varDelta T_0$ apply to the sidereal chronometer Kessels and Dent No. 1287. But in the exchange of longitude signals the mean-time chronometer Dent No. 2748 was employed, and its corrections were determined every evening, both before carrying it to, and after bringing it back from, the telegraph-office, by comparing it with No. 1287 by the method of coincidence of beats. The comparisons on each night, together with the resulting expressions for the corrections of No. 2748, are as follows:

April 12.—When No. 1287 indicated $7^{\text{h}}\ 53^{\text{m}}\ 12^{\text{s}}$ its correction was $+ 6^{\text{m}}\ 38^{\text{s}}.163 \pm 0^{\text{s}}.019$, and when it indicated $14^{\text{h}}\ 53^{\text{m}}\ 22^{\text{s}}$ its correction was $+ 6^{\text{m}}\ 37^{\text{s}}.582 \pm 0^{\text{s}}.018$. It was therefore gaining $0^{\text{s}}.0830$ per hour.

Chronometer Comparisons.

Before going to Telegraph-Office.						After returning from Telegraph-Office.					
No. 1287.			No. 2748.			No. 1287.			No. 2748.		
h.	m.	s.	h.	m.	s.	h.	m.	s.	h.	m.	s.
10	11	14.0 =	8	53	36.0	14	39	32.0 =	13	21	10.0
	14	20.5 =		56	42.0		42	39.0 =		24	16.5
	17	25.0 =		59	46.0		45	43.0 =		27	20.0

Hence, if T is the sidereal time at the meridian of the transit instrument, and T' the time indicated by No. 2748, we have

$$T = T' + 1^{\text{h}}\ 24^{\text{m}}\ 36^{\text{s}}.575 + 9^{\text{s}}.783\ (T' - 11^{\text{h}}) \pm 0^{\text{s}}.018$$

April 23.—When No. 1287 indicated $8^{\text{h}}\ 53^{\text{m}}\ 34^{\text{s}}$ its correction was $+ 6^{\text{m}}\ 25^{\text{s}}.988 \pm 0^{\text{s}}.033$; and when it indicated $8^{\text{h}}\ 53^{\text{m}}\ 40^{\text{s}}$ on April 26, its correction was $+ 6^{\text{m}}\ 19^{\text{s}}.786 \pm 0^{\text{s}}.057$. It was therefore gaining $0^{\text{s}}.0861$ per hour.

Chronometer Comparisons.

Before going to Telegraph-Office.						After returning from Telegraph-Office.					
No. 1287.			No. 2748.			No. 1287.			No. 2748.		
h.	m.	s.	h.	m.	s.	h.	m.	s.	h.	m.	s.
12	4	24.0 =	10	3	25.0	15	1	33.0 =	13	0	5.0
	7	29.5 =		6	30.0		4	35.0 =		3	6.5
	10	35.0 =		9	35.0		6	39.0 =		6	10.0

Hence

$$T = T' + 2^{\text{h}}\ 7^{\text{m}}\ 33^{\text{s}}.922 + 9^{\text{s}}.765\ (T' - 11^{\text{h}}) \pm 0.^{\text{s}}033$$

April 26.—When No. 1287 indicated $8^h\ 53^m\ 40^s$ its correction was $+\ 6^m\ 19^s.786 \pm 0^s.057$; and when it indicated $14^h\ 53^m\ 41^s$ its correction was $+\ 6^m\ 19^s.237 \pm 0^s.009$. It was therefore gaining $0^s.0915$ per hour.

Chronometer Comparisons.

Before going to Telegraph-Office.		After returning from Telegraph-Office.	
No. 1287.	No. 2748.	No. 1287.	No. 2748.
h. m. s.	h. m. s.	h. m. s.	h. m. s.
12 9 31.0 =	9 56 45.0	14 22 33.0 =	12 9 25.0
12 40.0 =	9 59 53.5	25 39.0 =	12 30.5
15 42.0 =	10 2 55.0	28 39.0 =	15 30.0

Hence

$$T = T' + 2^h\ 19^m\ 15^s.869 + 9.^s862\ (T' - 11^h) \pm 0.^s030$$

April 30.—When No. 1287 indicated $8^h\ 53^m\ 40^s$, on April 26, its correction was $+\ 6^m\ 19^s.786 \pm 0^s.057$; and when it indicated $8^h\ 53^m\ 48^s$, on April 30, its correction was $+\ 6^m\ 12^s.461 \pm 0^s.031$. It was therefore gaining $0^s.0763$ per hour.

Chronometer Comparisons.

Before going to Telegraph-Office.		After returning from Telegraph-Office.	
No. 1287.	No. 2748.	No. 1287.	No. 2748.
h. m. s.	h. m. s.	h. m. s.	h. m. s.
12 27 10.5 =	9 58 40.0	14 56 25.0 =	12 27 30.0
30 16.0 =	10 1 45.0	14 59 30.0 =	30 34.5
33 21.5 =	4 50.0	15 2 34.0 =	33 38.0

Hence

$$T = T' + 2^h\ 34^m\ 52^s.701 + 9^s.801\ (T' - 11^h) \pm 0^s.031$$

Collecting our results, we have the expressions given in Table V. for the corrections which must be applied to the time indicated by the face of the mean-time chronometer Dent No. 2748 in order to reduce it to sidereal time at the meridian of the transit instrument. T' is the time indicated by the chronometer at the instant for which the correction is required, and the quantities after the sign $\pm$ are approximately the probable errors.

TABLE V.—*Corrections to the Chronometer Dent No.* 2748.

Date.	Correction.
	h. m. s. s. h. s.
April 12	+ 1 24 36.575 + 9.783 (T' − 11.00) ± 0.018
23	+ 2 7 33.922 + 9.765 (T' − 11.00) ± 0.033
26	+ 2 19 15.869 + 9.862 (T' − 11.00) ± 0.030
30	+ 2 34 52.701 + 9.801 (T' − 11.00) ± 0.031

VI.—PERSONAL EQUATION.

In the beginning of August, 1870, Professor Eimbeck came to Washington, bringing with him his sidereal chronometer Kessels and Dent No. 1287, and his portable transit instrument C. S. No. 7. The latter was soon mounted on the collimator pier to the north of the transit circle and our relative personal equation was determined in the following manner: Professor Eimbeck and I made observations for time simultaneously, he using his own chronometer and portable transit instrument, and I using the transit circle, the Kessels clock, and chronograph. As far as possible we both employed the same stars. At the conclusion of each night's work he took my observing-key, and, by tapping upon it in coincidence with the beats of his chronometer, recorded upon the chronograph connected with the Kessels clock a series of signals similar to those which he sent from St. Louis when making telegraphic comparisons of time for difference of longitude. The correction necessary to reduce the local time determined by him to that determined by myself thus became known, and as his instrument and mine were in precisely the same meridian, this correction is evidently the required personal equation.

The observations for time made at Washington with the transit circle are given in Table VI, the arrangement of which is similar to that of Table II. The values of the constants employed during each night are as follows:

Date.	*c*	*b*	*a*
	s.	s.	s.
August 5	+ 0.04	− 0.21	− 0.02
8	+ .05	− .24	− .01
12	+ .05	− .23	− .03
15	0.00	− 0.27	− 0.24

TABLE VI.—*Transits of Stars Observed at Washington with the Transit Circle to determine the Corrections to the Kessels Sidereal Clock.*

Date.	Star.	Observer.	No. of Wires.	Time of Transit over Mean of Wires.	Cc	Bb	Aa	Correction for Instrument.	Corr. Transit.	Adopt'd Right Ascension.	Obs'd Clock Correction.	v
1870.				h. m. s.	s.	s.	s.	s.	s.	s.	s.	s.
Aug. 5	κ Ophiuchi . .	Ha.	9	16 52 15.60	+0.04	−0.19	−0 01	−0.16	15.44	32.49	−42.95	−.10
	d Herculis . .	Ha.	9	16 57 33.11	.05	.25	.00	.20	32.91	49.75	43.16	+.10
	μ^1 Sagittarii . .	Ha.	9	18 6 44.63	.04	0.11	− .02	0.09	44.54	1.49	43.05	.00
	δ Ursæ Minoris	Ha.	5	15 13.60	+ .67	+2.38	+ .25	−1.46	12.14	28.55	43.59	
	51 Cephei, S. P. .	Ha.	5	18 39 8.75	− .83	+2.56	− .33	+1.40	10.15	27.94	42.21	
	γ Aquilæ . .	Ha.	9	19 40 50.04	+ .04	−0.19	.01	−0.16	49.88	6.91	42.97	−.08
	β Aquilæ . .	Ha.	9	49 41.06	0.04	0.18	− .01	0.15	40.91	57.82	43.09	+.05
	λ Ursæ Minoris	Ha.	4	19 55 55.55	2.11	7.14	+ .81	4.22	51.33	12.58	38.75	
	α^2 Capricorni .	Ha.	9	20 11 35.89	+0.04	−0.13	−0.02	−0.11	35.78	52.72	−43.06	+.02
Aug. 8	η Herculis . .	Ha.	9	16 39 11.42	+0.06	−0.31	0.00	−0.25	11.17	27.72	−43.45	−.03
	κ Ophiuchi . .	Ha.	9	16 52 16.10	.05	0.21	.00	.16	15.94	32.45	43.49	+.01
	ε Ursæ Minoris	Ha.	5	17 0 9.59	.37	1.29	+ .05	.87	8.72	25.49	43.23	
	α^1 Herculis . .	Ha.	9	9 28.43	.05	0.23	.00	.18	28.25	44.81	43.44	−.04
	44 Ophiuchi . .	Ha.	9	17 19 11.53	.05	0.12	− .01	0.08	11.45	27.94	43.51	+.03
	δ Ursæ Minoris	S.	7	18 15 11.70	+0.84	−2.72	+ .12	−1.76	9.94	27.54	42.40	
	51 Cephei, S. P. .	S.	5	39 8.50	−1.03	+2.93	− .17	+1.73	10.23	29.17	41.06	
	β Lyræ . . .	S.	9	18 46 2.64	+0.06	−0.29	.00	−0.23	2.41	18.87	43.54	+.02
	κ Aquilæ . .	Ha.	9	19 30 39.67	.05	.17	− .01	.13	39.54	56.02	43.52	+.04
	γ Aquilæ . .	Ha.	9	40 50.52	.05	.21	.00	.16	50.36	6.91	43.45	−.03
	β Aquilæ . .	S.	9	49 41.48	0.05	0.20	− .01	0.16	41.32	57.82	43.50	−.02
	λ Ursæ Minoris	S.	4	19 55 58.82	+2.64	−8.16	+0.41	−5.11	53.71	10.65	−43.06	
Aug. 12	Polaris, S. P. .	S.	5	13 12 38.60	−2.07	+5.80	−0.98	+2.75	41.35	56.82	−44.53	
	α Virginis . .	F.	9	19 5.91	+0.05	−0.15	.02	−0.12	5.79	21.23	44.56	+.01
	ζ Virginis . .	F.	9	13 28 49.31	.05	.18	.02	.15	49.16	4.63	44.53	−.02
	ε Bootis. . .	F.	9	14 40 3.89	.06	.25	.01	.20	3.69	19.15	44.54	.01
	α^2 Libræ . . .	F.	9	44 26.97	.05	.14	− .03	.12	26.85	42.31	44.54	−.01
	β Bootis. . .	F.	9	14 57 48.23	.07	.30	.00	.23	48.00	3 37	44.63	+.08
	β Libræ . . .	F.	9	15 10 46.53	.05	.16	− .02	.13	46.40	1.87	44.53	−.02
	α^1 Herculis . .	Ha.	9	17 9 29.46	.05	.22	.01	.18	29.28	44.76	44.52	+.05
	α Ophiuchi . .	Ha.	9	29 40.32	.05	.21	.01	.17	40.15	55.68	44.47	.00
	μ Herculis . .	Ha.	9	17 42 8.61	.06	.26	.01	.21	8.40	23.93	44.47	.00
	μ^6 Sagittarii . .	Ha.	9	18 6 46.07	.05	0.12	− .03	0.10	45.97	1.44	44.53	+.06
	δ Ursæ Minoris	Ha.	5	15 11.65	.84	2.60	+ .37	1.39	10.21	26.22	43.99	
	1 Aquilæ . .	Ha.	9	28 54.62	.05	0.16	− .02	0.13	54.49	10.00	44.49	+.02
	α Lyræ . . .	Ha.	9	33 18.79	+0.06	−0.29	.00	−0.23	18.56	34.20	44.36	−.11
	51 Cephei, S. P. .	S.	5	18 39 13.22	−1.05	+2.81	− .50	+1.27	14.49	30.77	43.72	
	δ Aquilæ . .	S.	9	19 19 43.36	+0.05	−0.19	.02	−0.16	43.20	58.73	44.47	−.08
	κ Aquilæ . .	S.	9	30 40.76	.05	.16	.02	.13	40.63	56.02	44.61	+.06
	γ Aquilæ . .	S.	9	40 51.61	0.05	0.21	−0.01	0.17	51.44	6.90	44.54	−.01
	λ Ursæ Minoris	S.	5	19 55 56.26	2.64	7.82	+1.22	3.96	52.30	7.82	44.48	
	ε Delphini . .	S.	9	20 27 47.08	0.05	0.21	−0.01	0.17	46.91	2.36	44.55	.00
	1 Pegasi . .	S.	9	21 16 51.49	.05	.23	.01	.19	51.30	6.79	44.51	−.04
	β Aquarii . .	S.	9	21 25.29.87	+0.05	−0.16	−0.02	−0.13	29.74	45.16	−44.58	+.03

TABLE VI.—*Transits of Stars Observed at Washington, &c.*—Continued.

Date.	Star.	Observer.	No. of Wires.	Time of Transit over Mean of Wires.	Cc	Bb	Aa	Correction for Instrument.	Corr. Transit.	Adopt'd Right Ascension.	Obs'd Clock Correction.	v
1870.				h. m. s.	s.	s.	s.	s.	s.	s.	s.	s.
Aug. 15	α Ophiuchi . .	E.	9	17 29 41.47	0.00	−0.25	−0.11	−0.36	41.11	55.63	−45.48	+.03
	μ Herculis . .	Ha.	9	42 9.52	.00	.30	.05	.35	9.17	23.88	45.29	−.06
	γ² Sagittarii . .	E.	9	17 58 15.44	.00	.11	.26	.37	15.07	29.67	45.40	−.04
	μ¹ Sagittarii . .	Ha.	9	18 6 47.19	.00	0.15	−0.22	.37	46.82	1.41	45.41	+.07
	δ Ursæ Minoris	H.&E.	11	15 10.91	.00	3.06	+3.00	.06	10.85	25.30	45.55	
	I Aquilæ . .	Ha.	9	28 55.69	.00	0.19	−0.18	.37	55.32	9.98	45.34	.00
	α Lyræ . . .	E.	9	33 19.88	.00	−0.35	0.00	.35	19.53	34.15	45.38	−.05
	51 Cephei, S. P..	H.&E.	11	39 18.31	.00	+3.29	−4.01	.72	17.59	31.82	45.77	
	β Lyræ . . .	Ha.	9	18 46 4.49	.00	−0.32	0.03	.35	4.14	18.79	45.35	+.02
	ζ Aquilæ . .	E.	9	19 0 13.90	.00	.25	.11	.36	13.54	28.09	45.45	.02
	δ Aquilæ . .	Ha.	9	19 44.41	.00	.22	.14	.36	44.05	58.71	45.34	.01
	κ Aquilæ . .	E.	9	30 41.83	.00	.19	.17	.36	41.47	56.01	45.46	+.04
	γ Aquilæ . .	Ha.	9	40 52.53	.00	.24	.12	.36	52.17	6.89	45.28	−.04
	α Aquilæ . .	E.	9	19 45 14.37	0.00	−0.24	−0.12	−0.36	14.01	28.60	−45.41	−.01

Assuming $T_0 = 16^h\ 0^m$ by face of the Kessels clock, and $\theta = -\ 40^s.000$, the equations of condition, normal equations, and resulting values of $\delta\theta$ and R for each night are as follows:

WASHINGTON, AUGUST 5, 1870.

Equations of Condition.

$$0 = +\ 2^s.95 + \delta\theta + 0.87\ R$$
$$0 = +\ 3.16 + \delta\theta + 0.96\ R$$
$$0 = +\ 3.05 + \delta\theta + 2.11\ R$$
$$0 = +\ 2.97 + \delta\theta + 3.68\ R$$
$$0 = +\ 3.09 + \delta\theta + 3.83\ R$$
$$0 = +\ 3.06 + \delta\theta + 4.19\ R$$

Normal Equations.

$$0 = +\ 18.28 + \ \ 6.00\ \delta\theta + 15.64\ R$$
$$0 = +\ 47.62 + 15.64\ \delta\theta + 51.90\ R$$

Hence

$$\delta\theta = -\ 3^s.046$$
$$R = +\ 0.0004$$
$$\Delta T_0 = -\ 43.046 \pm 0^s.022$$

WASHINGTON, AUGUST 8, 1870.

On this evening observations were made by two persons, and therefore two values of ΔT_0 have been introduced in the equations.

Equations of Condition.

$$
\begin{aligned}
0 &= +\overset{s.}{3.45} + \delta\theta + 0 + 0.65\,R \\
0 &= +3.49 + \delta\theta + 0 + 0.87\,R \\
0 &= +3.44 + \delta\theta + 0 + 1.16\,R \\
0 &= +3.51 + \delta\theta + 0 + 1.32\,R \\
0 &= +3.54 + 0 + \delta\theta''' + 2.77\,R \\
0 &= +3.52 + \delta\theta + 0 + 3.51\,R \\
0 &= +3.45 + \delta\theta + 0 + 3.68\,R \\
0 &= +3.50 + 0 + \delta\theta''' + 3.83\,R
\end{aligned}
$$

Normal Equations.

$$
\begin{aligned}
0 &= +20.86 + 6.00\,\delta\theta + 0.00\,\delta\theta''' + 11.19\,R \\
0 &= +7.04 + 0.00\,\delta\theta + 2.00\,\delta\theta''' + 6.60\,R \\
0 &= +62.16 + 11.19\,\delta\theta + 6.60\,\delta\theta''' + 52.47\,R
\end{aligned}
$$

Hence

$$
\begin{aligned}
\delta\theta &= -\overset{s.}{3.475} \\
\delta\theta''' &= -3.518 \\
R &= -0.0011 \\
\varDelta T_0 &= -43.475 \pm \overset{s.}{0.010} \\
\varDelta T_0''' &= -43.518 \pm 0.014
\end{aligned}
$$

WASHINGTON, AUGUST 12, 1870.

On this evening observations were made by three persons, and therefore three values of the clock correction have been introduced in the equations.

Equations of Condition.

$$
\begin{aligned}
0 &= +\overset{s.}{4.56} + \delta\theta'' + 0 + 0 - 2.68\,R \\
0 &= +4.53 + \delta\theta'' + 0 + 0 - 2.52\,R \\
0 &= +4.54 + \delta\theta'' + 0 + 0 - 1.33\,R \\
0 &= +4.54 + \delta\theta'' + 0 + 0 - 1.26\,R \\
0 &= +4.63 + \delta\theta'' + 0 + 0 - 1.03\,R \\
0 &= +4.53 + \delta\theta'' + 0 + 0 - 0.82\,R \\
0 &= +4.52 + 0 + \delta\theta + 0 + 1.16\,R \\
0 &= +4.47 + 0 + \delta\theta + 0 + 1.49\,R \\
0 &= +4.47 + 0 + \delta\theta + 0 + 1.70\,R \\
0 &= +4.53 + 0 + \delta\theta + 0 + 2.11\,R \\
0 &= +4.49 + 0 + \delta\theta + 0 + 2.48\,R \\
0 &= +4.36 + 0 + \delta\theta + 0 + 2.56\,R \\
0 &= +4.47 + 0 + 0 + \delta\theta''' + 3.33\,R \\
0 &= +4.61 + 0 + 0 + \delta\theta''' + 3.51\,R \\
0 &= +4.54 + 0 + 0 + \delta\theta''' + 3.68\,R \\
0 &= +4.55 + 0 + 0 + \delta\theta''' + 4.46\,R \\
0 &= +4.51 + 0 + 0 + \delta\theta''' + 5.28\,R \\
0 &= +4.58 + 0 + 0 + \delta\theta''' + 5.42\,R
\end{aligned}
$$

Normal Equations.

$$0 = + 27.33 + 6.00\,\delta\theta'' + 0.00\,\delta\theta + 0.00\,\delta\theta''' - 9.64\,R$$
$$0 = + 26.84 \quad 0.00\,\delta\theta'' + 6.00\,\delta\theta + 0.00\,\delta\theta''' + 11.50\,R$$
$$0 = + 27.26 \quad 0.00\,\delta\theta'' + 0.00\,\delta\theta + 6.00\,\delta\theta''' + 25.68\,R$$
$$0 = + 124.18 - 9.64\,\delta\theta'' + 11.50\,\delta\theta + 25.68\,\delta\theta''' + 156.33\,R$$

Hence

$$\begin{aligned}
\delta\theta'' &= -\overset{s.}{4.554} \\
\delta\theta &= -4.474 \\
\delta\theta''' &= -4.545 \\
R &= +0.0005 \\
\varDelta T_0'' &= -44.554 \pm \overset{s.}{0.011} \\
\varDelta T_0 &= -44.474 \pm 0.017 \\
\varDelta T_0''' &= -44.545 \pm 0.014
\end{aligned}$$

Washington, August 15, 1870.

On this evening observations were made by two persons, and therefore two values of the clock correction have been introduced in the equations.

Equations of Condition.

$$\begin{aligned}
0 &= + \overset{s.}{5.48} + \delta\theta' + 0 + 1.50\,R \\
0 &= + 5.29 + 0 + \delta\theta + 1.70\,R \\
0 &= + 5.40 + \delta\theta' + 0 + 1.97\,R \\
0 &= + 5.41 + 0 + \delta\theta + 2.11\,R \\
0 &= + 5.34 + 0 + \delta\theta + 2.48\,R \\
0 &= + 5.38 + \delta\theta' + 0 + 2.56\,R \\
0 &= + 5.35 + 0 + \delta\theta + 2.77\,R \\
0 &= + 5.45 + \delta\theta' + 0 + 3.00\,R \\
0 &= + 5.34 + 0 + \delta\theta + 3.33\,R \\
0 &= + 5.46 + \delta\theta' + 0 + 3.51\,R \\
0 &= + 5.28 + 0 + \delta\theta + 3.68\,R \\
0 &= + 5.41 + \delta\theta' + 0 + 3.75\,R
\end{aligned}$$

Normal Equations.

$$\begin{aligned}
0 &= + 32.58 + 6.00\,\delta\theta' + 0.00\,\delta\theta + 16.29\,R \\
0 &= + 32.01 + 0.00\,\delta\theta' + 6.00\,\delta\theta + 16.07\,R \\
0 &= + 174.11 + 16.29\,\delta\theta' + 16.07\,\delta\theta + 93.86\,R
\end{aligned}$$

Hence

$$\begin{aligned}
\delta\theta' &= - \overset{s.}{5.465} \\
\delta\theta &= - 5.370 \\
R &= + 0.0129 \\
\varDelta T_0' &= - 45.465 \pm \overset{s.}{0.010} \\
\varDelta T_0 &= - 45.370 \pm 0.013
\end{aligned}$$

The observations for time made at Washington by Professor Eimbeck with the portable transit instrument C. S. No. 7 are given in Table VII, the arrangement of which is similar to that of Table IV. These observations were reduced by Professor Keith of the Coast Survey, who adopted, throughout the whole series, $R = -0^s.032$, and $c = +0^s.547$ with lamp west. The latter constant was determined from measures made with the right-ascension micrometer of the transit circle; that instrument being used as a collimator to the portable transit instrument. As already explained, some of the right ascensions employed by Professor Keith differed slightly from those used at this Observatory, and on that account a few small changes have been made in his reductions by Mr. Frisby and myself.

TABLE VII.—*Transits of Stars Observed at Washington by Professor William Eimbeck with the Portable Transit Instrument C. S. No. 7, to determine the Corrections to the Sidereal Chronometer Kessels and Dent No. 1287.*

Date.	Lamp.	No. of Wires.	Star.	Time of Transit over Mean of Wires.	b	Bb	Cc	r	Corr. Transit.	Adop'd Right Ascension.	Obs'd Chron. Correction.	v
1870.				h. m. s.	s.	s.	s.	s.	m. s.	m. s.	m. s.	s.
Aug. 5	E.	3	ε Ursæ Min. .	16 4 13.32	+0.09	+0.48	− 4.08	+0.03	4 9.75	59 25.97	+55 16.22	+0.76
	E.	7	44 Ophiuchi .	23 10.53	.06	.03	0.60	.02	23 9.98	18 27.97	17.99	.02
	E.	6	ω Draconis .	42 29.12	.07	.17	1.52	+ .01	42 27.78	37 45.31	17.53	+ .08
	E.	7	γ Draconis .	16 58 20.28	.04	.06	0.89	.00	58 19.45	53 37.33	17.88	− .65
	E.	5	μ¹ Sagittarii .	17 10 43.62	. .	0.04	0.59	.00	10 43.07	6 1.50	18.43	− .43
	E.	4	δ Ursæ Min. .	19 21.20	.11	+1.25	− 9.30	.00	19 13.15	14 28.44	15.29	+ .44
	E.	5	51 Cephei, S.P.	17 42 57.81	.11	−1.34	+11.40	− .02	43 7.85	38 28.06	20.21	+ .44
	E.	5	δ Aquilæ . .	18 23 41.14	. .	+0.09	− 0.55	.04	23 40.64	18 58.74	18.10	− .16
	E.	7	κ Aquilæ . .	34 38.39	.11	.08	.55	.04	34 37.88	29 56.03	18.15	.18
	E.	6	γ Aquilæ . .	44 49.43	. .	.10	.56	.05	44 48.92	40 6.92	18.00	.07
	E.	7	α Aquilæ . .	18 49 11.02	. .	.10	.55	.05	49 10.52	44 28.61	18.09	.16
	E.?	7	α² Capricorni.	19 15 35.19	+0.11	+0.07	− 0.57	−0.06	15 34.63	10 52.72	+55 18.09	−0.11
Aug. 8	E.	7	β¹ Scorpii . .	15 2 38.51	. .	+0.01	− 0.58	+0.06	2 38.00	57 54.34	+55 16.34	+0.10
	E.	6	τ Herculis .	20 36.03	+0.02	.03	.80	.05	20 35.31	15 51.24	15.93	− .08
	E.	7	ζ Ophiuchi .	34 45.89	+ .03	.02	.56	.04	34 45.39	30 1.69	16.30	+ .08
	E.	7	η Herculis .	43 12.50	. .	+ .02	.71	.04	43 11.85	38 27.72	15.87	.07
	E.	7	κ Ophiuchi .	15 56 16.83	. .	.00	.56	.04	56 16.31	51 32.45	16.14	.09
	E.	6	d Herculis .	16 1 34.50	. .	− .03	0.66	.03	1 33.84	56 49.70	15.86	.14
	E.	4	ε Ursæ Min. .	4 17.18	− .07	.38	4.05	.03	4 12.79	59 25.50	12.71	+ .46
	E.	7	α¹ Herculis .	12 29.16	.08	.08	0.56	.03	12 28.55	8 44.81	16.26	− .08
	E.	7	44 Ophiuchi .	23 11.96	. .	− .04	− 0.60	.03	23 11.35	18 27.94	16.59	.11
	E.	3	Gr'm. 966, S.P.	27 0.51	. .	+ .11	+ 2.10	.02	27 2.74	22 21.15	18.41	.54
	E.	6	α Ophiuchi .	33 40.08	.06	− .05	− 0.56	.02	33 39.49	28 55.72	16.23	− .04
	E.	6	ω Draconis .	42 31.92	.11	.26	− 1.52	+ .01	42 30.15	37 45.16	15.01	+0.18
	W.	7	γ Draconis .	16 58 20.79	.09	.14	+ 0.88	.00	58 21.53	53 37.25	15.72	−1.72
	W.	7	μ¹ Sagittarii .	17 10 44.36	. .	.03	0.59	.00	10 44.92	6 1.48	16.56	−0.11
	W.	3	δ Ursæ Min. .	19 10.24	.01	− .11	+ 9.24	.00	19 19.37	14 27.43	8.04	+1.10
	W.	4	51 Cephei, S.P.	17 43 15.74	− .04	+ .48	−11.34	− .01	43 4.87	38 29.28	24.41	+0.70
	W.	7	ζ Aquilæ . .	18 4 11.40	+ .01	.01	+ 0.56	.03	4 11.94	59 28.14	16.20	− .01
	W.	7	d Sagittarii .	14 46.86	. .	.01	.58	.03	14 47.42	10 3.95	16.53	+ .10
	W.	7	δ Aquilæ . .	23 41.88	.03	.02	.55	.04	23 42.41	18 58.74	16.33	− .06
	W.	7	κ Aquilæ . .	34 39.23	. .	.01	.55	.04	34 39.75	29 56.03	16.28	+ .12
	W.	7	γ Aquilæ . .	44 50.15	. .	.02	.56	.05	44 50.68	40 6.91	16.23	− .02
	W.	7	α Aquilæ . .	18 49 11.77	+0.02	+0.02	+ 0.55	−0.06	49 12.28	44 28.61	+55 16.33	−0.10
Aug. 12	E.	7	δ Ophiuchi .	15 12 20.33	−0.04	−0.03	− 0.55	+0.06	12 19.81	7 33.40	+55 13.59	+0.12
	E.	7	η Draconis .	27 3.49	− .01	− .02	1.16	.05	27 2.36	22 15.14	12.78	− .16
	E.	7	ζ Ophiuchi .	15 34 48.44	. .	.00	0.56	.05	34 47.93	30 1.61	13.68	−0.17
	E.	3	ε Ursæ Min. .	16 4 19.65	. .	+ .05	4.05	.03	59 15.68	59 24.86	9.18	+1.04
	E.	7	α¹ Herculis .	16 13 31.96	+0.03	+0.03	− 0.56	+0.03	13 34.46	8 44.75	+55 13.29	+0.03

TABLE VII.—*Transits of Stars Observed at Washington, &c.*—Continued.

Date.	Lamp.	No. of Wires.	Star.	Time of Transit over Mean of Wires.	*b*	*B b*	*C c*	*r*	Corr. Transit.	Adop'd Right Ascension.	Obs'd Chron. Correction.	*v*
1870.				h. m. s.	s.	s.	s.	s.	m. s.	m. s.	m. s.	s.
Aug. 12	E.	7	44 Ophiuchi .	16 23 14.74	+0.02	+0.01	− 0.61	+0.03	23 14.17	18 27.89	+55 13.72	−0.09
	E.	7	α Ophiuchi .	33 42.86	+ .03	+ .03	0.56	.02	33 42.35	28 55.68	13.33	+ .01
	E.	6	ω Draconis .	42 34.22	− .04	− .10	1.52	+ .01	42 32.61	37 44.96	12.35	−0.04
	E.	7	γ Draconis .	16 58 25.01	− .02	− .03	0.88	.00	58 24.10	53 37.15	13.05	−1.98
	E.	7	μ¹ Sagittarii .	17 10 48.43	. .	+ .01	0.59	.00	10 47.85	6 1.44	13.59	+0.01
	E.	3	δ Ursæ Min. .	19 29.99	+ .05	+ .57	− 9.24	.00	19 21.32	14 26.11	4.79	+1.30
	E.	2	51 Cephei, S.P.	42 56.32	+ .01	− .12	+11.34	− .02	43 7.52	38 30.78	23.26	−0.75
	E.	2	51 Cephei, S.P.	17 43 20.54	− .06	+ .73	−11.34	.02	43 9.91	38 30.78	20.87	+1.64
	W.	7	ζ Aquilæ . .	18 4 14.27	− .02	− .02	+ 0.56	.03	4 14.78	59 28.11	13.33	−0.08
	W.	7	δ Aquilæ . .	23 44.78	.00	.00	.55	.03	23 45.30	18 58.72	13.42	.01
	W.	7	κ Aquilæ . .	34 42.00	.00	.00	.55	.04	34 42.51	29 56.02	13.51	.02
	W,	7	γ Aquilæ . .	44 52.83	. .	.00	.56	.05	44 53.34	40 6.90	13.56	.20
	W.	7	α Aquilæ . .	49 14.50	. .	− .01	.55	.05	49 15.00	44 28.60	13.60	.23
	W.	7	β Aquilæ . .	18 53 43.83	. .	.01	.55	.05	53 44.32	48 57.81	13.49	.10
	W.	7	ε Delphini .	19 31 48.44	− .02	.02	.56	.07	31 48.91	27 2.36	13.45	.10
	W.	6	α Cygni . .	19 41 48.88	−0.02	−0.03	+ 0.77	−0.08	41 49.54	37 2.62	+55 13.08	−0.07
Aug. 15	W.	7	η Herculis .	15 43 16.34	−0.01	−0.01	+ 0.71	+0.04	43 17.08	38 27.58	+55 10.50	−0.10
	W.	7	κ Ophiuchi .	15 56 21.03	. .	+ .03	0.56	.03	56 21.65	51 32.35	10.70	− .09
	W.	3	ε Ursæ Min. .	16 4 12.34	+ .05	.26	4.05	.03	4 16.68	59 24.35	7.67	+ .28
	W.	7	44 Ophiuchi .	23 16.26	. .	.04	0.60	.03	23 16.93	18 27.85	10.92	+ .05
	W.	7	α Ophiuchi .	33 44.18	.14	.12	0.56	.02	33 44.88	28 55.61	10.73	− .08
	W.	7	ω Draconis .	42 33.24	.09	.22	1.52	+ .02	42 35.00	37 44.79	9.79	0.07
	W.	7	γ Draconis .	16 58 25.80	.08	.12	0.88	.00	58 26.80	53 37.07	10.27	2.00
	W.	7	μ¹ Sagittarii .	17 10 49.97	. .	.05	.59	.00	10 50.61	6 1.41	10.80	0.14
	W.	7	η Serpentis .	19 25.41	.11	+0.08	+ 0.55	.00	19 26.04	14 36.87	10.83	− .05
	W.	4	51 Cephei, S.P.	43 26.07	.12	−1.46	−11.34	− .01	43 13.26	38 31.84	18.58	+0.27
	E.	3	51 Cephei, S.P.	17 43 1.48	.20	−2.44	+11.34	.02	43 10.36	38 31.84	21.48	1.06
	E.	7	ζ Aquilæ . .	18 4 18.09	.25	+0.23	− 0.56	.02	4 17.74	59 28.09	10.35	0.08
	E.	7	d Sagittarii .	14 53.79	.25	.14	.58	.03	14 53.32	10 3.93	10.61	.08
	E.	7	δ Aquilæ . .	23 48.66	. .	.19	.55	.03	23 48.27	18 58.71	10.44	.08
	E.	7	κ Aquilæ . .	34 45.99	.22	.15	.55	.04	34 45.55	29 56.01	10.46	.20
	E.	7	γ Aquilæ . .	44 56.86	. .	.10	.56	.04	44 56.45	40 6.89	10.44	+ .10
	E.	7	α Aquilæ . .	49 18.52	. .	.10	.55	.05	49 18.09	44 28.59	10.50	− .02
	E.	7	β Aquilæ . .	18 53 47.92	.19	0.10	0.55	.05	53 47.47	48 57.80	55 10.33	+ .17
	E.	2	λ Ursæ Min. .	19 0 51.84	.10	3.41	29.05	.06	0 26.14	55 6.05	54 39.91	.58
	E.	7	μ Aquarii. .	50 30.75	. .	0.08	0.56	.08	50 30.19	45 40.78	55 10.59	.03
	E.	7	ν Cygni . .	19 57 12.74	+0.12	+0.16	− 0.72	−0.09	57 12.09	52 22.19	+55 10.10	+0.01

The adopted values of T_0 and θ for each night, together with the equations of condition, normal equations, and resulting values of $\delta\theta$ and a, are as follows:

WASHINGTON, AUGUST 5, 1870.

Equations of Condition.

$T_0 = 17^h\ 4^m\ 42^s$	$\theta = +\ 55^m\ 18^s.000$
$0 = +\ \overset{s.}{1.78} + \delta\theta -\ \ 5.120\ a$	$0 = -\ \overset{s.}{2.21} + \delta\theta + 16.860\ a$
$0 = + 0.01 + \delta\theta +\ \ 0.971\ a$	$0 = - 0.10 + \delta\theta +\ \ 0.584\ a$
$0 = + 0.47 + \delta\theta -\ \ 1.390\ a$	$0 = - 0.15 + \delta\theta +\ \ 0.724\ a$
$0 = + 0.12 + \delta\theta -\ \ 3.600\ a$	$0 = \ \ 0.00 + \delta\theta +\ \ 0.481\ a$
$0 = - 0.43 + \delta\theta +\ \ 0.923\ a$	$0 = - 0.09 + \delta\theta +\ \ 0.506\ a$
$0 = + 2.71 + \delta\theta - 12.520\ a$	$0 = - 0.09 + \delta\theta +\ \ 0.802\ a$

Normal Equations.

$$0 = +\ \ 2.02 + 12.00\ \delta\theta -\ \ 0.78\ a$$
$$0 = - 82.06 -\ \ 0.78\ \delta\theta + 485.89\ a$$

Hence

$$\delta\theta = -\ \overset{m.}{0}\ \ \overset{s.}{0.158}$$
$$a = +\ 0\ \ 0.169$$
$$\varDelta T_0 = +\ 55\ \ 17.842 \pm 0^s.080$$

WASHINGTON, AUGUST 8, 1870.

Equations of Condition.

$T_0 = 17^h\ 4^m\ 44^s$	$\theta = +\ 55^m\ 16^s.000$
$0 = -\ \overset{s.}{0.34} + \delta\theta + 0.899\ a$	$0 = +\ \overset{s.}{0.99} + \delta\theta -\ \ 1.390\ a$
$0 = + 0.07 + \delta\theta - 0.189\ a$	$0 = + 0.28 + \delta\theta -\ \ 3.600\ a$
$0 = - 0.30 + \delta\theta + 0.764\ a$	$0 = - 0.56 + \delta\theta +\ \ 0.923\ a$
$0 = + 0.13 + \delta\theta - 0.011\ a$	$0 = + 7.96 + \delta\theta - 12.520\ a$
$0 = - 0.14 + \delta\theta + 0.492\ a$	$0 = - 8.41 + \delta\theta + 16.860\ a$
$0 = + 0.14 + \delta\theta + 0.101\ a$	$0 = - 0.20 + \delta\theta +\ \ 0.433\ a$
$0 = + 3.29 + \delta\theta - 5.120\ a$	$0 = - 0.53 + \delta\theta +\ \ 0.894\ a$
$0 = - 0.26 + \delta\theta + 0.421\ a$	$0 = - 0.33 + \delta\theta +\ \ 0.584\ a$
$0 = - 0.59 + \delta\theta + 0.971\ a$	$0 = - 0.28 + \delta\theta +\ \ 0.724\ a$
$0 = - 2.41 + \delta\theta + 3.530\ a$	$0 = - 0.23 + \delta\theta +\ \ 0.481\ a$
$0 = - 0.23 + \delta\theta + 0.448\ a$	$0 = - 0.33 + \delta\theta +\ \ 0.506\ a$

Normal Equations.

$$0 = -\ \ \ 2.28 + 22.00\ \delta\theta +\ \ 6.20\ a$$
$$0 = - 272.33 +\ \ 6.20\ \delta\theta + 500.77\ a$$

Hence

$$\delta\theta = -\ \overset{m.}{0}\ \ \overset{s.}{0.050}$$
$$a = +\ 0\ \ 0.543$$
$$\varDelta T_0 = +\ 55\ \ 15.950 \pm \overset{s.}{0.071}$$

WASHINGTON, AUGUST 12, 1870.

Equations of Condition.

$T_0 = 17^h\ 4^m\ 47^s$	$\theta = +\ 55^m\ 13^s.000$
$0 = -\ 0^s.59 + \delta\theta + 0.676\,a$	$0 = -\ 10^s.26 + \delta\theta + 16.860\,a$
$0 = +\ 0.22 + \delta\theta - 0.836\,a$	$0 = -\ 7.87 + \delta\theta + 16.860\,a$
$0 = -\ 0.68 + \delta\theta + 0.764\,a$	$0 = -\ 0.33 + \delta\theta + 0.433\,a$
$0 = +\ 3.82 + \delta\theta - 5.120\,a$	$0 = -\ 0.42 + \delta\theta + 0.584\,a$
$0 = -\ 0.29 + \delta\theta + 0.421\,a$	$0 = -\ 0.51 + \delta\theta + 0.724\,a$
$0 = -\ 0.72 + \delta\theta + 0.971\,a$	$0 = -\ 0.56 + \delta\theta + 0.481\,a$
$0 = -\ 0.33 + \delta\theta + 0.448\,a$	$0 = -\ 0.60 + \delta\theta + 0.506\,a$
$0 = +\ 0.65 + \delta\theta - 1.390\,a$	$0 = -\ 0.49 + \delta\theta + 0.540\,a$
$0 = -\ 0.05 + \delta\theta - 3.600\,a$	$0 = -\ 0.45 + \delta\theta + 0.473\,a$
$0 = -\ 0.59 + \delta\theta + 0.923\,a$	$0 = -\ 0.08 + \delta\theta - 0.140\,a$
$0 = +\ 8.21 + \delta\theta - 12.520\,a$	

Normal Equations.

$$0 = -\ 11.92 + 21.00\,\delta\theta + 18.06\,a$$
$$0 = -\ 433.16 + 18.06\,\delta\theta + 772.36\,a$$

Hence

$$\delta\theta = +\ 0^m\ 0^s.087$$
$$a = +\ 0\ \ 0.559$$
$$\varDelta T_0 = +\ 55\ \ 13.087 \pm 0^s.109$$

WASHINGTON, AUGUST 15, 1870.

A preliminary reduction of the observations showed that there was something wrong about the adopted value of the collimation constant, and therefore a term involving a correction to it has been introduced in the equations of condition.

Equations of Condition.

$T_0 = 17^h\ 4^m\ 50^s$	$\theta = +\ 55^m\ 10^s.000$
$0 = -\ 0^s.50 + \delta\theta + 1.29\,\delta c - 0.011\,a$	$0 = -\ 0^s.35 + \delta\theta - 1.03\,\delta c + 0.433\,a$
$0 = -\ 0.70 + \delta\theta + 1.01\,\delta c + 0.492\,a$	$0 = -\ 0.61 + \delta\theta - 1.06\,\delta c + 0.894\,a$
$0 = +\ 2.33 + \delta\theta + 7.42\,\delta c - 5.120\,a$	$0 = -\ 0.44 + \delta\theta - 1.00\,\delta c + 0.584\,a$
$0 = -\ 0.92 + \delta\theta + 1.09\,\delta c + 0.971\,a$	$0 = -\ 0.46 + \delta\theta - 1.01\,\delta c + 0.724\,a$
$0 = -\ 0.73 + \delta\theta + 1.02\,\delta c + 0.448\,a$	$0 = -\ 0.44 + \delta\theta - 1.02\,\delta c + 0.481\,a$
$0 = +\ 0.21 + \delta\theta + 2.77\,\delta c - 1.390\,a$	$0 = -\ 0.50 + \delta\theta - 1.01\,\delta c + 0.506\,a$
$0 = -\ 0.27 + \delta\theta + 1.61\,\delta c - 3.600\,a$	$0 = -\ 0.33 + \delta\theta - 1.01\,\delta c + 0.540\,a$
$0 = -\ 0.80 + \delta\theta + 1.07\,\delta c + 0.923\,a$	$0 = +\ 30.09 + \delta\theta - 52.89\,\delta c - 40.680\,a$
$0 = -\ 0.83 + \delta\theta + 1.00\,\delta c + 0.663\,a$	$0 = -\ 0.59 + \delta\theta - 1.01\,\delta c + 0.747\,a$
$0 = -\ 8.58 + \delta\theta - 20.72\,\delta c + 16.860\,a$	$0 = -\ 0.10 + \delta\theta - 1.32\,\delta c - 0.046\,a$
$0 = -\ 11.48 + \delta\theta + 20.72\,\delta c + 16.860\,a$	

Normal Equations.

$$0 = + \quad 4.00 + 21.00\,\delta\theta - \quad 44.08\,\delta c - \quad 8.72\,a$$
$$0 = - 1634.97 - 44.08\,\delta\theta + 3738.42\,\delta c + 2102.63\,a$$
$$0 = - 1578.74 - \quad 8.72\,\delta\theta + 2102.63\,\delta c + 2270.36\,a$$

Hence

	m.	s.
$\delta\theta = +$	0	0.274
$\delta c = +$	0	0.102
$a = +$	0	0.602
$\varDelta T_0 = +$	55	10.274 $\pm 0^s.085$
$c = +$	0	0.649

Relative Personal Equation of Mr. Frisby and Professor Harkness.—If $\varDelta T_0$ represents the correction to the Kessels clock at any given instant, as determined by me; and $\varDelta T_0''$, the same correction as determined by Mr. Frisby; then the observations of April 26 give,

$$\varDelta T_0 = \varDelta T_0'' + 0.^s163$$

and those of August 12 give,

$$\varDelta T_0 = \varDelta T_0'' + 0^s.080$$

The mean is,

$$\varDelta T_0 = \varDelta T_0'' + 0^s.121$$

which I adopt.

Relative Personal Equation of Professors Eimbeck and Harkness.—In any case in which it is desired to determine personal equation, suppose that a comparison of the time-pieces of the senior and junior observers shows that when the face of the former indicates the time T_s, the face of the latter indicates the time T_j; and let $\varDelta T_s$ be the correction to the time-piece of the senior observer, as determined from his observations; $\varDelta T_j$, the correction to the time-piece of the junior observer, as determined from his observations; and m, the interval by which the junior observer notes the transit of an equatorial star later than the senior observer. Then, if $\varDelta T_s$ and $\varDelta T_j$ have been determined at the same meridian,

$$T_s + \varDelta T_s = T_j + \varDelta T_j + m$$

and hence

$$m = T_s - T_j + \varDelta T_s - \varDelta T_j$$

In the case of a single time-piece, if at any given instant its correction is $\varDelta T_s'$ from the observations of the senior observer, and $\varDelta T_j'$ from the observations of the junior observer, then

$$\varDelta T_s' = \varDelta T_j' + m$$

Considering all differences of longitude as essentially positive, if the senior observer occupies the

$\left\{\begin{matrix}\text{western}\\ \text{eastern}\end{matrix}\right\}$ station, m, taken with regard to its proper sign, must be $\left\{\begin{matrix}\text{added to}\\ \text{subtracted from}\end{matrix}\right\}$ the observed difference of longitude in order to free it from the effects of personal equation.

Designating by T_s the time indicated by the Kessels clock, and by T_j the time indicated by the chronometer Kessels and Dent No. 1287; the comparisons of time-pieces, and the computation of m from the observations given above are as follows:

Comparisons of Time-Pieces to determine Personal Equation.

Date.	No. of Signals.	T_s				T_j			$T_s - T_j$	
1870.		h.	m.	s.		h.	m.	s.	m.	s.
August 5	37	21	10	12.45	=	20	14	11.66	+ 56	0.79
8	37	20	10	11.04	=	19	14	11.66	55	59.38
12	37	21	2	9.25	=	20	6	11.66	55	57.59
15	37	20	4	7.30	=	19	8	11.66	+ 55	55.64

Each line in the column "*No. of Signals*" gives the number of signals read from the chronograph-sheet, the mean of which furnished the comparison recorded on the same line.

Computation of the Value of m.

Date.	ΔT_s		ΔT_j		$\Delta T_s - \Delta T_j$		$T_s - T_j$		m
1870.	m.	s.	m.	s.	m.	s.	m.	s.	s.
August 5	− 0	43.04	+ 55	17.76	− 56	0.80	+ 56	0.79	− 0.01
8	0	43.48	55	15.88	55	59.36	55	59.38	+ 0.02
12	0	44.47	55	12.99	55	57.46	55	57.59	0.13
15	− 0	45.32	+ 55	10.21	− 55	55.53	+ 55	55.64	+ 0.11

The values of m apparently divide themselves into two groups—the results of the first two nights agreeing with each other, and the results of the last two nights agreeing with each other. However, as there is no reason for supposing that the observations on one night are better than those on another, I have adopted the general mean, which is

$$m = + 0^s.062 \pm 0^s.025$$

As Professor Eimbeck occupied the western station during the exchange of longitude signals, this quantity must be subtracted from the observed difference of longitude in order to free it from personal equation.

VII.—EXCHANGE OF TIME-SIGNALS, AND RESULTING DIFFERENCE OF LONGITUDE.

The telegraph-line between Washington and St. Louis is made up entirely of wire stretched in the air, but, as it is 990 miles long, it cannot be worked in a single circuit. It was therefore divided into three circuits, and the signals were transmitted from each circuit to the next following by means of automatic repeaters, which were placed at Graf-

ton and Cincinnati. The number of statute-miles of wire, exclusive of that on the magnets, and the amount of battery, in each circuit, were as follows:

Washington to Grafton, 335 miles: at Washington, 60 modified Grove cells coupled up for quantity in two parallel series of 30 cells each; at Grafton, 50 Grove cells.

Grafton to Cincinnati, 310 miles: 50 Grove cells at Grafton; 64 Grove cells at Cincinnati.

Cincinnati to St. Louis, 345 miles: 62 Grove cells at Cincinnati; 60 Grove cells at St. Louis.

The signals employed in determining the difference of longitude were made by breaking a closed galvanic circuit; a method which seems best because the magnets used in telegraphing are much more certain to open promptly when the circuit is broken than to close promptly when it is re-established. This is true of a circuit including only a single magnet, but it applies with far greater force when, as in the present case, the signals are transmitted through several circuits by means of repeaters.

As the observer at St. Louis had neither a clock, a break circuit chronometer, nor a chronograph, it was necessary to employ other means in making the telegraphic comparisons of time-pieces there. The plan adopted is fully explained in the

PROGRAMME FOR THE EXCHANGE OF LONGITUDE-SIGNALS.

1. Local sidereal time will be determined at each station by observing transits of stars in the usual manner.

As the telegraph-wires do not extend to the observing-station at St. Louis, whenever signals are to be exchanged it will be necessary for the observer there to go to the telegraph-office in the Merchant's Exchange, carrying with him a *mean-time* box-chronometer beating half-seconds. The distance between the observing-station and the telegraph-office is about one and a half miles, and to avoid the chance of undetected tripping of the chronometer while being carried, it must be compared with the standard sidereal chronometer each evening immediately before starting from, and immediately after returning to, the observing-station.

2. Every afternoon the observer at St. Louis will notify the observer at Washington as to the state of the weather, and if it is clear at both places arrangements will be made to exchange signals in the evening. The hour of making the exchange will necessarily depend very much upon the convenience of the telegraph company, but it will usually be practicable to obtain the use of the wires some time between 10 p. m. and midnight.

3. The telegraph-office in the Merchant's Exchange at St. Louis, and the United States Naval Observatory at Washington, having been put in communication, the observer at the former will ask the observer at the latter if he is ready to receive signals, and, upon getting an affirmative reply, the St. Louis observer will wait until his *mean-time* chronometer indicates 50 seconds, and then he will send a rattle by means of his break-circuit key. This rattle will consist of ten or fifteen dots made at the rate of about five per second. At the beginning of the next minute he will commence sending his first series of signals. This will consist of thirteen taps on his key, made in exact coincidence with the beats of his chronometer at 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, and 0 seconds; a pause of five seconds, and a rattle. At the beginning of the next minute he will commence sending his second series of signals. This will consist of eleven taps on his key, made in coincidence with the beats of his chronometer at 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 seconds; a pause of five seconds, and a rattle. At the beginning of the next minute he will commence sending his third series of signals. This will consist of thirteen taps on his key, made in coincidence with the beats of his chronometer at 0, 10, 20, 30, 40, 50, 0, 10, 20, 30, 40, 50, and 0 seconds; a pause of five seconds, and a rattle. The three series of signals, from the beginning of the first rattle to the end of the last rattle, will occupy about four minutes and twenty seconds; and immediately upon their

completion the observer at Washington will notify the observer at St. Louis whether or not they have been properly received. If they have not been, they will be repeated; if they have been, the observer at St. Louis will telegraph to the observer at Washington the hour and minute indicated by the chronometer at the beginning of the first series. The observer at St. Louis will of course preserve a record of the hour and minute indicated by the chronometer at the commencement of *each* of the series of signals.

These signals will all be transmitted to Washington, where they will record themselves upon the chronograph-sheet along with the beats of the Washington clock, and, as the probable error of a single signal is only about $\pm 0^s.034$, they will furnish a very accurate comparison of time-pieces.

4. As soon as the observer at Washington has been notified of the hour and minute corresponding to the beginning of the first series of signals from St. Louis, he will ask the observer there if he is ready to receive signals from Washington, and upon receiving an affirmative reply he will connect the Washington *sidereal* clock with the telegraph-line in such a manner that its pendulum will break the circuit for somewhat less than one-tenth of a second every time it passes the central point of its arc, thus making a break once every second. In addition, the beginning of each minute will be marked by a double break—that is, by a break interpolated midway between the break corresponding to 0 seconds and that corresponding to 1 second.

These signals will be transmitted to St. Louis and there rendered audible by means of a sounder so adjusted that its *back* stroke is much louder than its forward one. Sitting beside this sounder, and keeping his eye upon his mean-time chronometer, the observer at St. Louis will wait until the *back* stroke[1] of the sounder coincides with the beat of the chronometer; and when this occurs he will note the time indicated by the latter, and also the time of arrival of the *next following* double break. Three such coincidences will be recorded, and, as they occur at intervals of about three minutes, the time required for so doing will not generally exceed twelve minutes. As soon as the double break following the third coincidence has been received, the observer at St. Louis will open the circuit and ask the observer at Washington what hour and minute of his clock corresponded to the break in question. The observer at Washington having furnished the desired information, the two stations will bid each other "good night," and this will close the exchanges for the evening.

N. B.—Although there will generally be an uncertainty of three or four seconds as to the exact instant when the beat of the sounder coincides with that of the chronometer, still special care must always be taken to note whether the coincidence occurs at a whole or at a half-second beat of the chronometer.

The record of signals received at St. Louis, from Washington, on the evening of April 12, together with their reduction, is appended in order to show how the comparison of time-pieces is deduced from the coincidences of beats observed in the manner just described.

Coincidence of breaks with beats of chronometer.			Next following double break.
h.	m.	s.	s.
12	17	45.0	32.0
	20	44.5	31.5
	23	40.0	31.0 = $14^h\ 42^m\ 0^s.0$ Wash. Clock.

As the double break corresponds to 0 seconds of the Washington clock, if from the seconds of the time of coincidence of beats, (increased by 60 when necessary,) the seconds of the time of the next following double break are subtracted, the remainder will be the seconds indicated by the face of the Washington clock at the time of the coincidence of beats. Thus, for the first coincidence recorded above,

$$45^s.0 - 32^s.0 = 13^s.0$$

[1] The back stroke is used because it corresponds to the break. The forward stroke corresponds to the subsequent closing of the circuit.

The last recorded coincidence and double break give, not only the seconds of the Washington clock, but the hour and minute also; thus furnishing the means of supplying the hour and minute to each of the other coincidences. Applying this process to the record given above, we obtain:

Kessels Clock.				Chronometer Dent No. 2748.		
h.	m.	s.		h.	m.	s.
14	35	13.0	=	12	17	45.0
	38	13.0	=		20	44.5
	41	9.0	=		23	40.0

If we let

$\varDelta\lambda$ = difference of longitude between two stations; west longitudes being taken as positive;

T_e = time by face of eastern clock when it sends a signal, and

T_w = time by face of western clock when that signal is received at the western station;

T'_w = time by face of western clock when it sends a signal, and

T'_e = time by face of eastern clock when that signal is received at the eastern station;

t = time occupied in the passage of a signal from one station to the other;

$\varDelta T_e$, $\varDelta T_w$, $\varDelta T'_e$, $\varDelta T'_w$ = respectively, the corrections necessary to reduce the times indicated by the faces of the eastern and western clocks to true local time at the instants T_e, T_w, T'_e, T'_w, then, neglecting personal equation, when the eastern clock sends and the signals are received at the western station, we have

$$\varDelta\lambda - t = (T_e - T_w) + (\varDelta T_e - \varDelta T_w)$$

and when the western clock sends, and the signals are received at the eastern station, we have

$$\varDelta\lambda + t = (T'_e - T'_w) + (\varDelta T'_e - \varDelta T'_w)$$

Hence

$$\varDelta\lambda = \frac{(T_e - T_w) + (T'_e - T'_w)}{2} + \frac{(\varDelta T_e - \varDelta T_w) + (\varDelta T'_e - \varDelta T'_w)}{2}$$

$$t = \frac{(T'_e - T'_w) - (T_e - T_w)}{2} + \frac{(\varDelta T'_e - \varDelta T'_w) - (\varDelta T_e - \varDelta T_w)}{2}$$

If the rates of the clocks are small, the second term in the expression for the value of t may usually be neglected.

The following are the results of the telegraphic comparisons of time-pieces, both at Washington and St. Louis, together with their reduction by means of the formulæ just given, used in connection with the data contained in the preceding pages. By way of explanation it is only necessary to remark that each line in the column headed "*No. of Signals*" gives the number of signals read off from the chronograph-sheet, the mean of which furnished the comparison recorded on the same line.

Comparisons of Time-Pieces obtained by reading off the Washington Chronograph-Sheets.

Date.	No. of Signals.	Kessels Clock at Washington.			Dent 2748 at St. Louis.			Means.						$T'_e - T'_w$.		
		h.	m.	s.	h.	m.	s.	h.	m.	s.	h.	m.	s.	h.	m.	s.
1870. April 12	13	14	3	53.22 =	11	46	30.00									
	11		5	27.94 =		48	4.50	14	5	35.00 =	11	48	11.50	+ 2	17	23.50
	13		7	23.83 =		50	0.00									
April 23	13	14	36	52.50 =	11	36	30.00									
	11		38	27.79 =		38	5.00	14	38	34.45 =	11	38	11.67	+ 3	0	22.78
	13		40	23.06 =		40	0.00									
April 26	13	14	37	34.28 =	11	25	30.00									
	11		39	9.62 =		27	5.00	14	39	16.26 =	11	27	11.67	+ 3	12	4.59
	13		41	4.88 =		29	0.00									
April 30	13	14	59	13.83 =	11	31	30.00									
	11	15	0	49.12 =		33	5.00	15	0	55.78 =	11	33	11.67	+ 3	27	44.11
	13		2	44.39 =		35	0.00									

Comparisons of Time-Pieces obtained by observing Coincidences of Beats at St. Louis.

Date.	No. of Coinci-dences.	Kessels Clock at Washington.			Dent 2748 at St. Louis.			Means.						$T_e - T_w$		
		h.	m.	s.	h.	m.	s.	h.	m.	s.	h.	m.	s.	h.	m.	s.
1870. April 12	1	14	35	13.00 =	12	17	45.00									
	1		38	13.00 =		20	44.50	14	38	11.67 =	12	20	43.17	+ 2	17	28.50
	1		41	9.00 =		23	40.00									
April 23	1	14	47	39.00 =	11	47	15.00									
	1		50	45.00 =		50	20.50	14	50	44.67 =	11	50	20.17	+ 3	0	24.50
	1		53	50.00 =		53	25.00									
April 26	1	14	22	22.00 =	11	10	20.50									
	1		25	27.00 =		13	25.00	14	25	27.33 =	11	13	25.33	+ 3	12	2.00
	1		28	33.00 =		16	30.50									
April 30	1	14	44	11.00 =	11	16	30.00									
	1		47	17.00 =		19	35.50	14	47	15.00 =	11	19	33.50	+ 3	27	41.50
	1		50	17.00 =		22	35.00									

Clock and Chronometer Corrections at the Times of the Exchange of Signals, computed by means of the Formulæ contained in Tables III and V.

Date.	Kessels Clock at Washington.	Dent 2748 at St. Louis.	$\Delta T'_e - \Delta T'_w$	$\Delta T_e - \Delta T_w$
1870.	h. m. s.	h. m. s.	h. m. s.	h. m. s.
April 12	− 0 0 1.995	+ 1 24 44.431	− 1 24 46.426	
	2.002	49.735		− 1 24 51.737
April 23	− 0 0 5.616	+ 2 7 40.138	− 2 7 45.754	
	5.623	42.114		− 2 7 47.737
April 26	− 0 0 7.188	+ 2 19 20.338	− 2 19 27.526	
	7.179	18.075		− 2 19 25.254
April 30	− 0 0 8.929	+ 2 34 58.122	− 2 35 7.051	
	8.926	55.896		− 2 35 4.822

Observed Values of the Difference of Longitude between Washington and St. Louis, and of the Time occupied in the passage of a Galvanic Signal between those Cities.

Date.	$\frac{1}{2}(T'_e - T'_w)$	$\frac{1}{2}(T_e - T_w)$	$\frac{1}{2}(\Delta T'_e - \Delta T'_w)$	$\frac{1}{2}(\Delta T_e - \Delta T_w)$	$\Delta\lambda$	t
1870.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	s.
April 12	+ 1 8 41.750	+ 1 8 44.250	− 0 42 23.213	− 0 42 25.868	+ 0 52 36.919	+ 0.155
23	1 30 11.390	1 30 12.250	1 3 52.877	1 3 53.868	36.895	.131
26	1 36 2.295	1 36 1.000	1 9 43.763	1 9 42.627	36.905	.159
30	+ 1 43 52.055	+ 1 43 50.750	− 1 17 33.526	− 1 17 32.411	+ 0 52 36.868	+ 0.190

The time occupied in the passage of a signal between the two cities seems to have varied considerably on different nights; probably owing to slight changes in the adjustments of the repeaters connecting the several circuits. Of course these observations give no clew to the speed of a galvanic wave propagated in a continuous conductor.

Taking the mean of the values of λ, we find that the observing-station at St. Louis is

$$0^h\ 52^m\ 36^s.897 \pm 0^s.007$$

west of the transit circle at Washington. This result requires two small corrections: 1st, for personal equation, Professor Eimbeck observing the transit of an equatorial star $0^s.062 \pm 0^s.025$ later than myself; and, 2d, to refer the difference of longitude to the center of the dome of this Observatory, that point being 77.8 feet $= 0^s.066$ east of the transit circle. Making these corrections, and having regard to the probable error of the personal equation, our final result is that

The Observing-Station at St. Louis, in the Washington University grounds, on St. Charles street, between Seventeenth and Eighteenth streets, is west of the center of the Dome of the United States Naval Observatory at Washington

$$0^h\ 52^m\ 36^s.90 \pm 0^s.026$$

Very respectfully,

WM. HARKNESS,
Professor of Mathematics, U. S. Navy.

Rear-Admiral B. F. Sands, U. S. N.,
Superintendent U. S. Naval Observatory, Washington, D. C.

WASHINGTON OBSERVATIONS FOR 1870.—APPENDIX II.

REPORTS

ON

OBSERVATIONS OF ENCKE'S COMET

DURING ITS RETURN IN 1871.

BY

ASAPH HALL AND WM. HARKNESS,
PROFESSORS OF MATHEMATICS, U. S. NAVY.

PREPARED AT THE U. S. NAVAL OBSERVATORY
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REPORT

OF

PROFESSOR ASAPH HALL, U. S. N.

REPORT OF PROFESSOR HALL, U. S. N.

United States Naval Observatory,
Washington, January 15, 1872.

Sir: The motion of Encke's comet around the sun has given rise to many interesting investigations, and the results obtained are such as will doubtless lead, sooner or later, to a new and complete discussion of the observations, together with a new investigation of the theory of the comet. The following observations were made during the favorable return of the comet in 1871. They are divided into two parts. The first part consists of determinations of position, and in order to give a clear idea of the degree of accuracy attained, these observations are given in detail. The second part comprises the notes on the appearance of the comet in the telescope, and to these are added four drawings, made from sketches of the comet on October 17, November 17, and December 1 and 2. These drawings are intended to illustrate the changes of form that the comet underwent as it approached the sun.

§ 1.

The following observations of the position of the comet were made with the filar micrometer of the Equatorial of the Naval Observatory. The reticule of this micrometer consists of three parallel wires, with equatorial intervals of $12^s.4$, for observations of the difference of right ascension; and perpendicular to these are five parallel wires used for measuring differences of declination. The differences of right ascension were observed by means of a chronograph, the pen of which is worked by a sidereal clock. This clock is situated on the first floor of the Observatory, and for the sake of convenience, the time of the observation was taken from a sidereal chronometer, the second of which was compared with the clock on the chronograph-sheet, at the beginning and end of each observation. No attention is paid to the clock further than to keep its rate so small that the effect of the rate on the observed difference of right ascension is always insensible. In reading off the chronograph-sheet, the zero is assumed to be at the second of the clock nearest the zero second of the chronometer. This assumption introduces an error into the determination of the time of the observation, which will vary from $0^s.0$ to $0^s.5$; and, in the case of a comet or swiftly-moving body, a correction for the error of the zero is combined with the correction of the chronometer. The chronometer was compared on each night with the Kessels clock, the standard clock of the Observatory.

In what follows $\Delta\alpha$ and $\Delta\delta$ are the observed differences of right ascension and declination at the corresponding chronometer-time. The value of $\Delta\alpha$ is generally the

mean of three wires, except in cases when the star and comet were so situated that it was inconvenient to observe all the wires, when only two wires were used. The value of $\Delta\delta$ is the result of a single bisection with the micrometer wire. The quantities under the head *Red.* are the reductions of the observed $\Delta\alpha$ and $\Delta\delta$ to the assumed chronometer-time of the observation, which is designated by *c*. These reductions have been computed from the ephemeris of the comet given by S. von Glasenapp, Astronomische Nachrichten, No. 1854, after the ephemeris had been corrected so as to represent very nearly the apparent motion of the comet. $\Delta_0\alpha$ and $\Delta_0\delta$ are the valuse of $\Delta\alpha$ and $\Delta\delta$ reduced to the chronometer time *c;* Δc is the correction of the chronometer on sidereal time; and $\Delta\rho$ is the correction for differential refraction. $r(\alpha)$ denotes the probable error of a difference of right ascension depending on an observation over a single wire, and $r(\delta)$ the probable error of a single bisection in declination. In the statement of the number of comparisons, the first number indicates the number of wires observed in right ascension, and the last the number of bisections in declination.

The positions of the stars of comparison are those which have been computed from the catalogues, no "systematic corrections" having been applied. The positions have been brought forward to 1871.0 by means of Peter's value of the constant of precession, and the reductions of the stars to apparent place have been computed from the quantities given in the American Ephemeris. These reductions are given for each date below the adopted mean position of the star.

It should be stated that in a few of the earlier observations, or until November 17, the point observed was the center of the figure of the comet, while in the later observations the nucleus of the comet was observed.

The objective of the Equatorial has an aperture of 9.6 inches, and a magnifying power of 132 was used in all the observations.

November 2.

Chron. Time.	$\Delta\alpha$	Red.	$\Delta_0\alpha$	Chron. Time.	$\Delta\delta$	Red.	$\Delta_0\delta$
h. m. s.	m. s.	s.	s.	h. m. s.	″	″	″
21 53 40.8	− 0 48.75	− 1.30	50.05	22 2 48.0	− 167.2	+ 7.2	− 160.0
55 18.7	− 0 48.75	− 0.63	49.38	4 50.0	− 181.3	+ 9.7	− 171.6
56 55.6	− 0 51.10	+ 0.03	51.07	6 27.0	− 189.4	+ 11.6	− 177.8
58 25.0	− 0 51.20	+ 0.64	50.56	8 9.0	− 188.2	+ 13.7	− 174.5
59 57.2	− 0 51.45	+ 1.27	50.18	9 55.0	− 189.4	+ 15.8	− 173.6

No. of Comparisons, (10.5.)

h. m. s.	m. s. s.	′ ″ ″
$c =$ 21 56 51.5	$\Delta_0\alpha = -$ 0 50.25 ± 0.19	$\Delta_0\delta = -$ 2 51.5 ± 2.1
$\Delta c = -$ 4 28.3	$\Delta\rho =$ 0.00	$\Delta\rho = -$ 0.0
	$r(\alpha) = \pm\ 0^s.61$	$r(\delta) = \pm\ 4''.6$

Star of Comparison, a.

Authority.	α	δ
	h. m. s.	° ′ ″
Bessel	22 37 4.28	+ 37 7 41.1
Washington observations, (2) .	4.06	41.3
Adopted	22 37 4.13	+ 37 7 41.2
Reduction	+ 1.95	+ 23.9

NOVEMBER 6.

Chron. Time.	$\Delta\alpha$	Red.	$\Delta_0\alpha$	Chron. Time.	$\Delta\delta$	Red.	$\Delta_0\delta$
h. m. s.	m. s.	s.	s.	h. m. s.	″	″	″
22 34 8.4	+ 0 49.73	− 6.56	43.17	23 43 46.0	− 442.4	− 11.7	− 454.1
36 10.6	+ 0 50.67	− 5.71	44.96	46 0.0	− 443.1	− 7.4	− 450.5
37 59.4	+ 0 50.23	− 4.96	45.27	48 15.0	− 459.6	− 3.1	− 462.7
39 41.0	+ 0 49.33	− 4.25	45.08	50 27.0	− 454.6	+ 1.2	− 453.4
41 28.7	+ 0 48.47	− 3.50	44.97	53 36.0	− 455.2	+ 5.3	− 449.9
58 49.3	+ 0 40.76	+ 3.75	44.51	56 10.0	− 468.2	+ 12.1	− 456.1
23 0 21.0	+ 0 40.00	+ 4.39	44.39				
1 45.3	+ 0 39.40	+ 4.98	44.38				
3 18.3	+ 0 38.30	+ 5.63	43.93				
4 45.3	+ 0 38.63	+ 6.23	44.86				

No. of Comparisons, (30.6.)

h. m. s.	m. s. s.	′ ″ ″
$c =$ 22 49 50.7	$\Delta_0\alpha = +$ 0 44.55 ± 0.15	$\Delta_0\delta = -$ 7 34.4 ± 1.3
$\Delta c = -$ 4 34.3	$\Delta\rho =$ 0.00	$\Delta\rho = -$ 0.0
	$r(\alpha) = \pm\ 0^s.79$	$r(\delta) = \pm\ 3''.1$

Star of Comparison, b.

Authority.	α	δ
	h. m. s.	° ′ ″
Bessel	21 55 23.61	+ 34 40 54.7
Reduction	+ 1.54	+ 23.4

November 7.

Chron. Time.	Δ α	Red.	Δ₀ α	Chron. Time.	Δ δ	Red.	Δ₀ δ
h. m. s.	m. s.	s.	s.	h. m. s.	"	"	"
22 23 23.2	− 0 30.00	− 4.79	34.79	22 30 28.0	− 60.9	− 9.2	− 70.1
24 55.4	− 0 28.90	− 4.15	33.05	34 11.0	− 75.7	− 1.4	− 77.1
26 13.0	− 0 29.85	− 3.61	33.46	35 40.0	− 73.9	+ 1.7	− 72.2
27 28.3	− 0 30.10	− 3.08	33.18	37 18.0	− 68.2	+ 5.1	− 63.1
28 41.3	− 0 29.85	− 2.58	32.43	38 49.0	− 69.0	+ 8.3	− 60.7
40 54.7	− 0 35.95	+ 2.52	33.43				
42 14.7	− 0 37.70	+ 3.07	34.63				
43 36.1	− 0 36.20	+ 3.64	32.56				
44 57.1	− 0 37.55	+ 4.20	33.35				
46 19.2	− 0 39.45	+ 4.77	34.68				

No. of Comparisons, (20.5.)

h. m. s.	m. s. s.	′ ″ ″
c = 22 34 52.3	$\Delta_0 \alpha$ = − 0 33.56 ± 0.19	$\Delta_0 \delta$ = − 1 8.6 ± 2.0
Δc = − 4 35.4	$\Delta \rho$ = 0.00	$\Delta \rho$ = − 0.0
	$r(\alpha) = \pm\ 0^s.83$	$r(\delta) = \pm\ 4''.5$

Star of Comparison, c.

Authority.	α	δ
	h. m. s.	° ′ ″
Bessel	21 46 47.91	+ 33 46 30.2
Reduction	+ 1.46	+ 23.1

November 8.

Chron. Time.	Δ α	Red.	₀ α	Chron. Time.	Δ δ	Red.	Δ₀ δ
h. m. s.	m. s.	s.	s.	h. m. s.	"	"	"
22 56 54.0	+ 0 21.45	− 4.31	17.14	23 4 19.5	+ 275.4	− 6.7	+ 268.7
58 5.4	+ 0 22.40	− 3.82	18.58	6 7.1	+ 273.4	− 2.7	+ 270.7
59 38.2	+ 0 21.65	− 3.18	18.47	9 26.3	+ 254.3	+ 4.8	+ 259.1
23 0 55.1	+ 0 21.20	− 2.65	18.55	11 15.1	+ 257.5	+ 8.9	+ 266.4
1 48.8	+ 0 19.50	− 2.28	17.24	12 50.3	+ 244.7	+ 12.5	+ 257.2
15 18.5	+ 0 14.50	+ 3.33	17.83				
16 33.8	+ 0 14.45	+ 3.84	18.29				
17 38.2	+ 0 13.30	+ 4.28	17.58				
18 52.6	+ 0 13.15	+ 4.79	17.94				

No. of Comparisons, (18.5.)

h. m. s.	m. s. s.	′ ″ ″
$c = 23\ 7\ 18.3$	$\Delta_0 \alpha = +\ 0\ 17.96 \pm 0.12$	$\Delta_0 \delta = +\ 4\ 24.4 \pm 1.8$
$\Delta c = -\ 4\ 36.2$	$\Delta \rho =\ 0.00$	$\Delta \rho = +\ 0.1$
	$r(\alpha) = \pm\ 0^s.52$	$r(\delta) = \pm\ 4''.0$

Star of Comparison, d.

Authority.	α	δ
	h. m. s.	° ′ ″
Washington observations, (3) .	21 35 47.62	+ 32 47 16.3
Reduction	+ 1.36	+ 22.7

November 10.

Chron. Time.	$\Delta \alpha$	Red.	$\Delta_0 \alpha$	Chron. Time.	$\Delta \delta$	Red.	$\Delta_0 \delta$
h. m. s.	m. s.	s.	s.	h. m. s.	″	″	″
22 13 10.3	− 2 50.00	− 5.33	55.33	22 13 10.3	+ 146.9	− 33.8	+ 113.1
17 48.4	− 2 49.05	− 3.45	52.50	17 48.4	+ 124.6	− 21.9	+ 102.7
21 54.9	− 2 49.95	− 1.79	51.74	21 54.9	+ 132.8	− 11.3	+ 121.5
26 7.3	− 2 52.75	− 0.08	52.83	26 7.3	+ 129.6	− 0.5	+ 129.1
30 45.8	− 2 50.75	+ 1.80	48.95	30 45.8	+ 102.8	+ 11.4	+ 114.2
35 5.2	− 2 55.10	+ 3.55	51.55	35 5.2	+ 92.6	+ 22.5	+ 115.1
39 23.0	− 2 56.55	+ 5.30	51.25	39 23.0	+ 82.0	+ 33.6	+ 115.6

No. of Comparisons, (14.7.)

h. m. s.	m. s. s.	′ ″ ″
$c = 22\ 26\ 19.3$	$\Delta_0 \alpha = -\ 2\ 52.02 \pm 0.49$	$\Delta_0 \delta = +\ 1\ 55.9 \pm 2.1$
$\Delta c = -\ 4\ 38.7$	$\Delta \rho =\ 0.00$	$\Delta \rho = +\ 0.0$
	$r(\alpha) = \pm\ 1^s.46$	$r(\delta) = \pm\ 5''.4$

Star of Comparison, e.

Authority.	α	δ
	h. m. s.	° ′ ″
Bessel	21 19 38.87	+ 30 55 13.2
Reduction	+ 1.21	+21.7

November 11.

Chron. Time.	$\Delta\alpha$	Red.	$\Delta_\circ\alpha$	Chron. Time.	$\Delta\delta$	Red.	$\Delta_\circ\delta$
h. m. s.	m. s.	s.	s.	h. m. s.	$''$	$''$	$''$
22 11 57.0	+ 3 32.03	− 7.41	24.62	22 11 57.0	+ 196.5	− 50.2	+ 146.3
18 43.7	+ 3 29.45	− 4.70	24.75	18 43.7	+ 192.9	− 31.9	+ 161.0
24 34.3	+ 3 26.13	− 2.36	23.77	24 34.3	+ 176.5	− 16.0	+ 160.5
32 1.8	+ 3 22.20	+ 0.62	22.82	32 1.8	+ 151.5	+ 4.2	+ 155.7
36 56.6	+ 3 21.23	+ 2.58	23.81	36 56.6	+ 140.8	+ 17.5	+ 158.3
41 38.2	+ 3 19.03	+ 4.46	23.49	41 38.2	+ 130.4	+ 30.2	+ 160.6
46 17.2	+ 3 17.60	+ 6.32	23.92	46 17.2	+ 114.3	+ 42.8	+ 157.1

No. of Comparisons, (20.7.)

	h. m. s.	m. s. s.	° $'$ $''$
c =	22 30 29.0	$\Delta_\circ\alpha = + 3$ 23.88 ± 0.17	$\Delta_\circ\delta = + 2$ 37.1 ± 1.3
Δc =	− 4 40.2	$\Delta\rho =$ 0.00	$\Delta\rho = +$ 0.0
		$r(\alpha) = \pm\ 0^s.76$	$r(\delta) = \pm\ 3''.5$

Star of Comparison, f.

Authority.	α	δ
	h. m. s.	° $'$ $''$
Bessel	21 3 41.58	+ 29 51 10.9
Reduction	+ 1.08	+ 20.8

November 17.

Chron. Time.	$\Delta\alpha$	Red.	$\Delta_\circ\alpha$	Chron. Time.	$\Delta\delta$	Red.	$_\circ\delta$
h. m. s.	m. s.	s.	s.	h. m. s.	$''$	$''$	$''$
23 1 57.4	− 1 6.70	− 2.53	9.23	23 1 57.4	+ 65.6	− 23.5	+ 42.1
5 10.5	− 1 7.50	− 1.40	8.90	5 10.5	+ 60.4	− 12.9	+ 47.5
7 52.6	− 1 8.53	− 0.44	8.93	7 52.6	+ 54.9	− 4.1	+ 50.8
10 34.9	− 1 10.40	+ 0.52	9.88	10 34.9	+ 34.0	+ 4.8	+ 38.8
13 15.3	− 1 10.43	+ 1.46	8.97	13 15.3	+ 32.4	+ 13.5	+ 45.9
15 52.3	− 1 11.00	+ 2.39	8.61	15 52.3	+ 29.7	+ 22.1	+ 51.8

No. of Comparisons, (18.6.)

	h. m. s.	m. s. s.	$'$ $''$ $''$
c =	23 9 7.2	$\Delta_\circ\alpha = - 1$ 9.09 ± 0.12	$\Delta_\circ\delta = + 0$ 46.2 ± 1.4
Δc =	− 4 49.7	$\Delta\rho =$ 0.00	$\Delta\rho = +$ 0.0
		$r(\alpha) = \pm\ 0^s.50$	$r(\delta) = \pm\ 3''.4$

Star of Comparison, g.

Authority.	α	δ
	h. m. s.	° ′ ″
Bessel	20 13 51.38	+ 22 35 40.7
Reduction	+ 0.74	+16.1

NOVEMBER 25.

Chron. Time.	$\Delta\alpha$	Red.	$\Delta_0\alpha$	Chron. Time.	$\Delta\delta$	Red.	$\Delta_0\delta$
h. m. s.	m. s.	s.	s.	h. m. s.	″	″	″
22 38 57.6	− 2 29.30	− 3.19	32.49	22 38 57.6	+ 291.5	− 36.8	+ 254.7
43 35.0	− 2 30.73	− 1.83	32.56	43 35.0	+ 280.3	− 21.2	+ 259.1
48 6.9	− 2 32.27	− 0.51	32.78	48 6.9	+ 264.4	− 5.9	+ 258.5
52 3.3	− 2 33.70	+ 0.64	33.06	52 3.3	+ 242.2	+ 7.4	+ 249.6
56 1.8	− 2 34.93	+ 1.80	33.13	56 1.8	+ 235.5	+ 20.8	+ 256.3
60 24.0	− 2 36.23	+ 3.08	33.15	60 24.0	+ 220.2	+ 35.6	+ 255.8

No. of Comparisons, (18.6.)

h. m. s.	m. s. s.	′ ″ ″
c = 22 49 51.4	$\Delta_0\alpha$ = − 2 32.86 ± 0.08	$\Delta_0\delta$ = + 4 15.7 ± 0.9
Δc = − 5 0.6	$\Delta\rho$ = 0.00	$\Delta\rho$ = + 0.1
	$r(\alpha) = \pm\, 0^s.35$	$r(\delta) = \pm\, 2''.3$

Star of Comparison, h.

Anthority.	α	δ
	h. m. s.	° ′ ″
Bessel	19 13 49.17	+ 11 47 51.0
Reduction	+ 0.54	+ 8.9

NOVEMBER 25.

Chron. Time.	$\Delta\alpha$	Red.	$\Delta_0\alpha$	Chron. Time.	$\Delta\delta$	Red.	$\Delta_0\delta$
h. m. s.	m. s.	s.	s.	h. m. s.	″	″	″
22 48 6.9	− 1 3.67	− 1.76	5.43	22 48 6.9	− 211.4	− 20.4	− 231.8
52 3.3	− 1 5.67	− 0.61	6.28	52 3.3	− 231.9	− 7.1	− 239.0
56 1.8	− 1 6.67	+ 0.55	6.12	56 1.8	− 243.7	+ 6.3	− 237.4
60 24.0	− 1 8.03	+ 1.83	6.20	60 24.0	− 254.4	+ 21.1	− 233.3

No. of Comparisons, (12.4.)

h. m. s.	m. s. s.	′ ″ ″	
$c = 22\ 54\ 9.0$	$\Delta_{\circ}\alpha = -1\ 6.01 \pm 0.13$	$\Delta_{\circ}\delta = -3\ 55.4 \pm 1.1$	
$\Delta c = -\ 5\ 0.6$	$\Delta\rho\ \ 0.00$	$\Delta\rho = -\ 0.1$	
	$r(\alpha) = \pm 0^{s}.45$	$r(\delta) = \pm 2''.3$	

Star of Comparison, i.

Authority,	α	δ
	h. m. s.	° ′ ″
Bessel	19 12 21.01	+ 11 55 47.8
Reduction	+ 0.50	+ 8.9

November 26.

Chron. Time.	$\Delta\alpha$	Red.	$\Delta_{\circ}\delta$	Chron. Time.	$\Delta\delta$	Red.	$\Delta_{\circ}\delta$
h. m. s.	m. s.	s.	s.	h. m. s.	″	″	″
22 21 51.3	− 0 54.77	− 1.81	56.58	22 21 51.3	− 475.7	− 21.3	− 497.0
24 14.0	− 0 55.80	− 1.13	56.93	24 14.0	− 477.6	− 13.3	− 490.9
26 49.4	− 0 56.97	− 0.39	57.36	26 49.4	− 497.1	− 4.6	− 501.7
29 14.2	− 0 57.70	+ 0.30	57.40	29 14.2	− 504.5	+ 4.4	− 500.1
32 3.5	− 0 58.07	+ 1.10	56.97	32 3.5	− 517.9	+ 12.9	− 505.0
34 59.7	− 0 58.97	+ 1.94	57.03	34 59.7	− 527.7	+ 22.8	− 504.9

No. of Comparisons, (17.6.)

h. m. s.	m. s. s.	′ ″ ″
$c = 22\ 28\ 12.0$	$\Delta_{\circ}\alpha = -0\ 57.05 \pm 0.08$	$\Delta_{\circ}\delta = -8\ 19.9 \pm 1.5$
$\Delta c = -\ 5\ 1.8$	$\Delta\rho =\ \ 0.00$	$\Delta\rho = -\ 0.2$
	$r(\alpha) = \pm 0^{s}.35$	$r(\delta) = \pm 3''.6$

Star of Comparison, k.

Authority.	α	δ	Wt.
	h. m. s.	° ′ ″	
Bessel	19 5 24.56	+ 10 41 11.8	1
Lamont	24.84	7.0	1
Adopted	19 5 24.70	+ 10 41 9.4	
Reduction	+ 0.49	+ 8.1	

NOVEMBER 27.

Chron. Time.	$\Delta\,\alpha$	Red.	$\Delta_0\,\alpha$	Chron. Time.	$\Delta\,\delta$	Red.	$\Delta_0\,\delta$
h. m. s.	m. s.	s.	s.	h. m. s.	"	"	"
22 28 27.8	+ 0 4.67	− 2.40	2.27	22 33 19.0	+ 113.1	− 12.3	+ 100.8
29 9.6	+ 0 4.30	− 2.20	2.10	34 41.9	+ 106.0	− 7.8	+ 98.2
29 55.0	+ 0 4.30	− 1.99	2.31	36 2.7	+ 105.7	− 3.3	+ 102.4
30 55.3	+ 0 3.80	− 1.71	2.09	37 37.3	+ 92.9	+ 2.0	+ 94.9
31 33.2	+ 0 3.90	− 1.53	2.37	39 26.0	+ 89.4	+ 8.0	+ 97.4
42 25.7	+ 0 0.77	+ 1.51	2.28	41 1.9	+ 86.9	+ 13.3	+ 100.2
43 13.0	+ 0 0.53	+ 1.73	2.26				
44 1.9	+ 0 0.60	+ 1.96	2.56				
44 57.6	+ 0 0.30	+ 2.22	2.52				
45 38.8	+ 0 0.00	+ 2.41	2.41				

No. of Comparisons, (30.6.)

	h. m. s.	m. s. s.	′ ″ ″
	$c = 22\ 37\ 1.8$	$\Delta_0\,\alpha = +\,0\ \ 2.32 \pm 0.03$	$\Delta_0\,\delta = +\,1\ \ 39.0 \pm 0.7$
	$\Delta c = -\ \ 5\ \ 3.9$	$\Delta\,\rho = \ \ 0.00$	$\Delta\,\rho = +\ \ 0.0$
		$r\,(\alpha) = \pm\,0^{s}.17$	$r\,(\delta) = \pm\,1''.8$

Star of Comparison, *l*.

Authority.	α	δ	Wt.
	h. m. s.	° ′ ″	
Bessel	18 57 36.64	+ 9 10 52.7	1
Lamont	36.44	56.5	1
Lalande	36.64	59.9	½
Adopted.	18 57 36.56	+ 9 10 55.7	
Reduction	+ 0.48	+ 7.2	

NOVEMBER 29.

Chron. Time.	$\Delta\,\alpha$	Red.	$\Delta_0\,\alpha$	Chron. Time.	$\Delta\,\delta$	Red.	$\Delta_0\,\delta$
h. m. s.	m. s.	s.	s.	h. m. s.	"	"	"
22 49 47.8	+ 2 42.13	− 3.08	39.05	22 49 47.8	+ 189.3	− 37.5	+ 151.8
53 51.3	+ 2 40.70	− 1.99	38.71	53 51.3	+ 174.6	− 24.2	+ 150.4
59 7.2	+ 2 39.40	− 0.58	38.82	59 7.2	+ 157.9	− 7.1	+ 150.8
23 4 22.2	+ 2 38.33	+ 0.83	39.16	23 4 22.2	+ 135.6	+ 10.0	+ 145.6
8 20.9	+ 2 37.43	+ 1.89	39.32	8 20.9	+ 122.3	+ 23.0	+ 145.3
12 14.6	+ 2 36.37	+ 2.94	39.31	12 14.6	+ 112.8	+ 35.7	+ 148.5

No. of Comparisons, (18.6.)

h. m. s.	m. s. s.	′ ″ ″
$c = 23\ 1\ 17.3$	$\Delta_0 a = +2\ 39.06 \pm 0.07$	$\Delta\delta = +2\ 28.7 \pm 0.8$
$\Delta c = -\ 5\ 7.4$	$\Delta\rho = 0.00$	$\Delta\rho = +\ 0.1$
	$r(a) = \pm 0^s.29$	$r(\delta) = \pm 1''.9$

Star of Comparison, m.

Authority.	a	δ	Wt.
	h. m. s.	° ′ ″	
Bessel	18 41 46.34	+ 6 31 8.8	1
Lamont	45.84	10.8	1
Schjellerup	46.10	9.4	2
Adopted	18 41 46.10	+ 6 31 9.6	
Reduction	+ 0.46	+ 5.4	

December 1.

Chron. Time.	Δa	Red.	$\Delta_0 a$	Chron. Time.	δ	Red.	$\Delta_0 \delta$
h. m. s.	m. s.	s.	s.	h. m. s.	″	″	″
22 36 10.3	+ 2 39.25	− 2.63	36.62	22 36 10.3	+ 169.9	− 32.4	+ 137.5
40 6.3	+ 2 37.67	− 1.61	36.06	40 6.3	+ 158.8	− 19.9	+ 138.9
44 16.5	+ 2 37.03	− 0.54	36.49	44 16.5	+ 144.2	− 6.7	+ 137.5
48 13.3	+ 2 35.97	+ 0.48	36.45	48 13.3	+ 134.6	+ 5.9	+ 140.5
52 55.8	+ 2 34.40	+ 1.69	36.09	52 55.8	+ 124.5	+ 20.9	+ 145.4
56 51.0	+ 2 33.83	+ 2.70	36.53	56 51.0	+ 112.2	+ 33.3	+ 145.5

No. of Comparisons, (17.6.)

h. m. s.	m. s. s.	′ ″ ″
$c = 22\ 46\ 22.2$	$\Delta_0 a = +2\ 36.37 \pm 0.07$	$\Delta_0 \delta = +2\ 20.9 \pm 1.0$
$\Delta c = -\ 5\ 8.5$	$\Delta\rho = -\ 0.01$	$\Delta\rho = +\ 0.1$
	$r(a) = \pm 0^s.28$	$r(\delta) = \pm 2''.5$

Star of Comparison, n.

Authority.	a	δ	Wt.
	h. m. s.	° ′ ″	
Lamont	18 29 18.07	+ 3 57 56.2	1
Reduction	+ 0.46	+ 4.0	

December 2.

Chron. Time.	Δ α	Red.	Δ₀ α	Chron. Time.	Δ δ	Red.	Δ₀ δ
h. m. s.	m. s.	s.	s.	h. m. s.	″	″	″
22 39 45.8	− 0 8.63	− 2.11	10.74	22 43 29.0	− 300.2	− 14.6	− 314.8
40 43.0	− 0 8.73	− 1.87	10.60	44 48.4	− 307.3	− 10.5	− 317.8
41 40.0	− 0 9.13	− 1.63	10.76	46 1.8	− 313.3	− 6.6	− 319.9
42 42.0	− 0 9.13	− 1.37	10.50	47 23.6	− 316.6	− 2.3	− 318.9
51 39.5	− 0 11.35	+ 0.89	10.46	48 48.3	− 318.9	+ 2.1	− 316.8
52 44.6	− 0 11.80	+ 1.16	10.64	50 7.4	− 325.2	+ 6.2	− 319.0
53 40.4	− 0 11.95	+ 1.40	10.55				
54 40.6	− 0 12.05	+ 1.64	10.41				
55 38.2	− 0 12.60	+ 1.89	10.71				

No. of Comparisons, (18.6.)

$c = 22^h\ 48^m\ 8^s.2$ $\Delta_0 \alpha = -0^m\ 10^s.60 \pm 0^s.03$ $\Delta_0 \delta = -5'\ 17''.9 \pm 0''.5$

$\Delta c = -\ 5^m\ 9^s.0$ $\Delta \rho = +\ 0^s.02$ $\Delta \rho = -\ 0''.3$

$r(\alpha) = \pm\ 0^s.13$ $r(\delta) = \pm\ 1''.2$

Star of Comparison, o.

Authority.	α	δ	Wt.
	h. m. s.	° ′ ″	
Bessel	18 25 58.43	+ 2 49 59.1	1
Schjellerup	58.50	53.7	2
Lamont	58.55	56.4	1
Adopted	18 25 58.50	+ 2 49 55.7	
Reduction	+ 0.47	+ 3.5	

December 5.

Chron. Time.	Δ α	Red.	Δ₀ α	Chron. Time.	Δ δ	Red.	Δ₀ δ
h. m. s.	m. s.	s.	s.	h. m. s.	″	″	″
23 6 8.6	− 1 43.27	− 1.51	44.78	23 6 8.6	+ 207.0	− 19.2	+ 187.8
9 31.6	− 1 44.10	− 0.71	44.81	9 31.6	+ 184.9	− 9.1	+ 175.8
12 37.3	− 1 44.67	+ 0.02	44.65	12 37.3	+ 183.5	+ 0.2	+ 183.7
15 44.2	− 1 45.20	+ 0.75	44.45	15 44.2	+ 172.0	+ 9.5	+ 181.5
18 44.2	− 1 44.87	+ 1.44	43.43	18 44.2	+ 160.1	+ 18.6	+ 178.7

No. of Comparisons, (15.5.)

h. m. s.	m. s. s.	′ ″ ″
$c = 23$ 12 33.2	$\Delta_0 a = -$ 1 44.42 ± 0.18	$\Delta_0 \delta = +$ 3 1.5 ± 1.4
$\Delta c = -$ 5 11.8	$\Delta \rho = +$ 0.04	$\Delta \rho = +$ 0.6
	$r(a) = \pm\ 0^s.69$	$r(\delta) = \pm\ 3''.1$

Star of Comparison, *p.*

Authority.	a	δ	Wt.
	h. m. s.	° ′ ″	
Bessel	18 9 54.60	− 1 0 7.8	1
Schjellerup	54.25	7.1	2
Lamont	54.44	5.1	1
Adopted	18 9 54.38	− 1 0 6.8	
Reduction	+ 0.49	+ 1.8	

December 7.

Chron. Time.	Δa	Red.	$\Delta_0 a$	Chron. Time.	$\Delta \delta$	Red.	$\Delta_0 \delta$
h. m. s.	m. s.	s.	s.	h. m. s.	″	″	″
22 57 27.9	− 2 56.87	− 1.10	57.97	22 57 27.9	− 171.8	− 14.2	− 186.0
23 2 37.4	− 2 59.87	+ 0.06	59.81	23 2 37.4	− 190.8	+ 0.8	− 190.0
6 59.3	− 2 60.70	+ 1.04	59.66	6 59.3	− 203.6	+ 13.4	− 190.2

No. of Comparisons, (9.3.)

h. m. s.	m. s. s.	′ ″ ″
$c = 23$ 2 21.5	$\Delta_0 \delta = -$ 2 59.15 ± 0.40	$\Delta_0 \delta = -$ 3 8.7 ± 0.9
$\Delta c = -$ 5 11.9	$\Delta \rho = +$ 0.05	$\Delta \rho = -$ 0.6
	$r(a) = \pm\ 1^s.19$	$r(\delta) = \pm\ 1''.6$

Star of Comparison, *q.*

Authority.	a	δ	Wt.
	h. m. s.	° ′ ″	
Lamont	18 0 9.07	− 3 14 56.9	1
Schjellerup	9.38	48.8	2
Lalande	9.58	48.8	1
Adopted	18 0 9.35	− 3 14 50.8	
Reduction	+ 0.51	+ 0.9	

Collecting the results, we have the following positions of the comet. The quantities given under the head *log* $p \times \Delta$ are the logarithms of the parallactic coefficients, computed with a value of the solar parallax equal to 8''.85—the coefficient for α being expressed in seconds of time:

Date.		Washington M. T.			α			$\log p \times \Delta$	δ				$\log p \times \Delta$
1871.		h.	m.	s.	h.	m.	s.			°	'	''	
November	2	7	5	6	22	36	15.83	9.0402n	+	37	5	13.6	9.5130
	6	7	42	7	21	56	9.70	9.0745	+	34	33	43.7	9.8559
	7	7	23	14	21	46	15.81	9.0225	+	33	45	44.7	9.9203
	8	7	51	38	21	36	6.94	9.3058	+	32	52	3.5	0.0678
	10	7	2	52	21	16	48.06	9.1754	+	30	57	30.8	0.1248
	11	7	3	3	21	7	6.54	9.2518	+	29	54	8.8	0.1913
	17	7	17	50	20	12	43.03	9.5302	+	22	36	43.0	0.4996
	25	6	26	59	19	11	16.85	9.5765	+	11	52	15.6	0.6592
	25	6	31	16	19	11	15.50	9.5824	+	11	52	1.3	0.6613
	26	6	1	27	19	4	28.14	9.5521	+	10	32	57.6	0.6643
	27	6	6	17	18	57	39.36	9.5738	+	9	12	41.9	0.6813
	29	6	22	33	18	44	25.62	9.6151	+	6	33	43.8	0.7103
December	1	5	59	48	18	31	54.89	9.6109	+	4	0	21.2	0.7242
	2	5	57	37	18	25	48.39	9.6178	+	2	44	41.0	0.7306
	5	6	10	7	18	8	10.49	9.6472	—	0	57	2.9	0.7448
	7	5	52	5	17	57	10.76	9.6482	—	3	17	59.2	0.7498

Comparing the observations with the ephemeris of S. von Glasenapp, the following residuals are found, taken in the sense of the computed minus the observed place of the comet:

Date.		In α.			In δ.		
			m.	s.		'	''
November	2	—	0	57.28	+	18	37.3
	6	—	1	37.18	+	16	51.8
	7	—	1	45.50	+	16	1.0
	8	—	1	56.04	+	15	17.6
	10	—	2	18.07	+	13	19.3
	11	—	2	19.98	+	12	0.6
	17	—	2	58.55	+	3	24.2
	25	—	3	17.78	—	6	34.0
	25	—	3	17.65	—	6	34.3
	26	—	3	18.35	—	7	25.7
	27	—	3	18.39	—	8	23.2
	29	—	3	17.28	—	9	42.4
December	1	—	3	15.35	—	10	46.9
	2	—	3	13.74	—	11	15.3
	5	—	3	7.15	—	12	1.0
	7	—	3	1.78	—	11	57.2

§ 2.

The following are the notes and remarks on the search for the comet, and its appearance after discovery:

September 8.—A careful search was made for the comet, but nothing could be seen in the place predicted by S. von Glasenapp. The sweep was extended a degree or more on each side of the computed declination, and through several degrees in right ascension.

September 21.—Looked for Encke's comet after the moon set, and made a careful search, but saw nothing.

October 9.—Looked carefully for the comet but could not see it. It appears, however, from subsequent observations, that on this night I did not extend the search far enough in declination.

October 11.—On this night a faint object was found with the comet-seeker by Paymaster H. P. Tuttle, U. S. N.; but on account of the bad seeing and the distance of the object from the predicted place, it was not certainly recognized to be the comet. An observation of the 17th proved it to be so.

October 17.—The comet was found early in the evening, as soon as the twilight was gone. It was very faint, without nucleus, and was a loose, disjointed-looking object. It appeared to be about 5′ or 6′ in diameter, with a vague and indistinct outline. No attempt was made to obtain an accurate observation for position, but an instrumental position gave the following corrections to Glasenapp's ephemeris:

$$\Delta\alpha = -\ 38^{s} \qquad\qquad \Delta\delta = -\ 13'$$

October 18.—The comet was examined at 10 o'clock. It was very faint and diffuse, and no trace of a nucleus could be seen. On this night the comet passed over a star of the 11.12th magnitude. The light of the star did not appear to be sensibly dimmed by its passage through the comet.

An instrumental position confirmed the corrections of the ephemeris found on the preceding night.

November 2.—Determinations of position were begun on this day. The comet had no nucleus, and was very difficult to observe.

November 6.—The comet was still very difficult to observe, but there appeared to be a little condensation at the center.

November 7.—The comet difficult to observe. It was thought to be a little elongated.

November 11.—The comet is a little brighter to-night. It appears elongated in the direction of the sun.

November 17.—The comet decidedly easier to observe. It is elongated in the direction of the sun, and has a fan-like appearance. On the following side of the comet there is a condensation, and, apparently, the beginning of a nucleus.

November 18.—Observed the spectrum of the comet in Professor Harkness's spectroscope. There were two bands visible. One of these bands was quite bright; the other a faint one.

November 25.—The comet has more condensation than hitherto. The nucleus is on the following side, the comet being elongated in the direction of the sun.

November 29.—The comet is brighter to-night and better to observe. The nucleus is on the following side of the comet.

December 1.—The nucleus is on the following side of the comet. The coma is almost wholly on the preceding side of the nucleus, and there is scarcely a trace of any on the following side. To-night the matter of the coma appears to issue from the nucleus toward the preceding side and to fall over each way, like water in the jet of a fountain. The comet was barely visible to the naked eye at 7 o'clock.

December 2.—The comet was visible to the naked eye at 6^h 30^m. The coma is mostly on the preceding side of the nucleus, and appears to flow out and fall over in curves. On the north preceding side there is a very bright part of the coma. The coma is a little on the following side of the nucleus to-night, but there it is very faint.

December 5.—The comet visible to the naked eye, and the telescope was set on it directly. It is diffuse, and the observation for position a little uncertain. The comet is so near the horizon that no very good idea can be got of its form. The coma, however, appears to be more on the following side of the nucleus than heretofore, although the most of it is still on the preceding side.

December 7.—The comet was visible to the naked eye at 6^h 10^m. In the telescope it looked fuzzy and blurred so near the horizon. The observation for position was hurried, and is, therefore, uncertain.

I first saw this comet in 1858, and have observed it at four returns. My impression is that it has been fainter during its present return than I have ever seen it before, considering its distance from the earth and sun. At this return, however, the comet approached the sun at such a time that it was quite near the earth when at a considerable distance from the sun, and in the case of this comet the formula assumed for computing the intensity of the light may not be exactly correct.

The changes of form that this comet undergoes as it approaches the sun and recedes from it have been observed at many of its returns. The general fact appears to be that there is a condensation in the matter of the comet and the formation of a nucleus as it approaches its perihelion, and an expansion of the matter and disappearance of the nucleus as the comet recedes from the sun. This appearance was observed by Struve in 1828, and his micrometrical measurements put beyond doubt the reality of the phenomenon. To an observer who sees such changes going on the question will naturally occur whether his determinations of position made at different times and when the comet has such different forms are strictly comparable. It would appear that this should be carefully considered in any discussion of the motion of the comet.

DESCRIPTION OF THE DRAWINGS.

The drawings of the comet have been placed on the page in such a way that the top of the page represents the *preceding* direction, the bottom of the page the *following* direction, the right-hand side of the page the *south*, and the left-hand side the *north*.

On October 17 the comet was very nearly circular in form, with a vague and indistinct outline. It was nearly of a uniform brightness over its whole surface, and there was no indication of a nucleus.

On November 17 the comet was elongated, the nucleus or condensed part of the comet being on the extreme following side. At this time the nucleus did not appear to be the center of action in the matter of the comet, as it appeared on December 1 and 2.

On December 1 the nucleus of the comet was a round, well-defined spot, and under a power of 132 appeared to be about 25″ or 30″ in diameter. The matter of the comet appeared to flow out from this nucleus toward the preceding side, falling over in parabolic curves about equally toward the north and south. There was very little of the coma on the following side of the nucleus. My observation extended over half an hour, and I noticed no change in the form of the comet.

On December 2 the nucleus of the comet appeared as on the 1st, and the matter of the comet appeared to flow from it in a similar manner; but on this day by far the larger portion of the coma was on the north preceding side of the nucleus. The coma appeared more on the following side of the nucleus than at any time heretofore. A line passing through the nucleus and indicating the direction of the boundary of the coma on the following side would make an angle of about 15° with a circle of declination. The curved part of the coma on the north preceding side of the nucleus was very bright.

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REPORT

OF

PROFESSOR WM. HARKNESS, U. S. N.

REPORT OF PROFESSOR HARKNESS, U. S. N.

UNITED STATES NAVAL OBSERVATORY,
Washington, February 1, 1872.

SIR: I have the honor to submit to you the following report in relation to a series of spectroscopic and polariscopic observations of Encke's Comet, made by me during the latter part of November and the beginning of December, 1871.

I.—DESCRIPTION OF INSTRUMENTS, AND METHOD OF OBSERVATION.

The *Telescope* employed was my equatorially-mounted portable achromatic refractor, of 3.01 inches clear aperture and 43.58 inches focus, made by Alvan Clark & Sons, of Cambridgeport, Massachusetts.

The *Spectroscope* was a single-prism instrument, which is fully described in my report on the total solar eclipse of August 7, 1869.[1] Its optical constants are as follows:

Small Telescope:	
Focal distance of object-glass	6.55 inches.
Clear aperture of object-glass	0.86 inch.
Diameter of field of view	5° 33′
Magnifying power	5.71 diameters.
Collimating Lens for Slit:	
Focal distance	6.52 inches.
Clear aperture	0.82 inch.
Collimating Lens for Micrometer:	
Focal distance	4.17 inches.
Clear aperture	0.82 inch.
Prism:	
Refracting angle	60° 8′
Dispersion from A to H_2	4° 5′
Minimum deviation of line D	47° 44′
Refractive index	1.613
Density	3.532

The magnifying power of this spectroscope, when used in connection with the 43½-inch telescope, is 38 diameters. The apparent length of the spectrum, as seen in the small telescope, is 23° 19′; or, in other words, it appears as a spectrum 4.12 inches long viewed at a distance of ten inches. Under favorable circumstances the line D is seen distinctly double.

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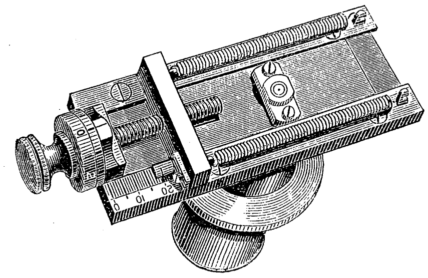
[1] Washington Observations for 1867, Appendix II, pp. 27-30.

As the spectrum of Encke's Comet was very faint, the photographed micrometer-scale of the spectroscope was removed, and the micrometer, shown in Fig. 1, was inserted in its place. This consists essentially of a micrometer-screw, which moves a brass plate pierced with a hole 0.00796 of an inch in diameter. The light passing through this hole is reflected from the surface of the prism and appears in the field of view of the spectroscope-telescope as a bright disk, with an apparent diameter of 36′ 55″, which can be made to traverse the whole length of the spectrum by turning the micrometer-screw. The illumination of the disk can be adjusted to the brightness of the spectrum under observation with the greatest nicety. If it is required to be very brilliant, the direct light of a lantern may be thrown into the hole; a less degree of brightness may be secured by passing the light through a piece of ground glass; and, finally, the luminosity may be varied down to absolute invisibility by reflecting the light into the hole from the back of the observer's hand held at a suitable angle. This last plan was employed in the case of the comet. The micrometer-head is half an inch in diameter, and divided to one-tenth of a revolution, while each complete revolution of the screw moves the brass plate 0.0181 of an inch, which corresponds to an angular distance of 14′ 40″.5. The apparent motion of the luminous disk in the field of view of the spectroscope-telescope is, therefore, 1° 23′ 48″ for each revolution of the screw, or 50″.3 for the hundredth part of a revolution; and the limit of reading on the micrometer-head is thus not far from the limit of visibility in the telescope.

Fig. 1.



In using this micrometer, the readings on the line whose place was to be determined were habitually made alternately with readings on a sodium-line produced by the flame of an alcohol-lamp with a salted wick held before the object-glass of the large telescope. The measures are thus entirely differential, and there is no risk of errors having been introduced by undetected changes of zero. It has been found convenient to assume that the reading of the sodium-line is always exactly 10.00 revolutions, and to apply the difference between this assumed reading and the actual reading, at any time, as a correction to all readings on other lines made at that time. From these corrected readings the wave-lengths of the corresponding lines can at once be deduced by means of a table which was constructed from the following observations of the principal lines in the solar spectrum:

TABLE I.—*Observations of Fraunhofer's Lines to determine the value of the Spectroscope Micrometer-Screw.*

Line.	Wave-Length.	Micrometer Reading.					
		Nov. 21.	Nov. 22.	Nov. 22.	Nov. 26.	Jan. 15.	Mean.
a	718.5	. .	. .	7.00	6.99	. .	7.00
B	686.7	. .	7.63	7.60	7.56	7.61	7.60
C	656.2	8.22	. .	. .	8.21	8.25	8.23
α	627.7	. .	. .	. .	. .	8.94	8.94
D	589.2	10.00	10.00	10.00	10.00	10.00	10.00
E	526.9	12.39	. .	12.36	12.35	12.36	12.36
b	517.4	12.81	. .	. .	12.78	12.81	12.80
F	486.1	. .	14.54	14.50	14.47	14.49	14.50
G	430.7	. .	. .	18.72	18.72	18.70	18.71
h	410.1	. .	. .	. .	20.87	20.91	20.89
H_1	396.8	. .	. .	22.57	22.55	22.57	22.56
H_2	393.3	. .	. .	23.08	23.03	23.11	23.07

The wave-lengths, which are expressed in millionths of a millimeter, are taken from Ångström's Atlas; and the observed micrometer readings have been corrected so as to make the reading of the sodium-line always 10.00 revolutions. The numbers contained in the second and eighth columns were employed to construct an interpolating curve, in the manner pointed out by Professor Pickering.[1] For that purpose a pair of rectangular co-ordinates were laid down; and then making vertical distances represent micrometer readings, and horizontal distances the squares of the reciprocals of the wave-lengths, a number of points were obtained, which necessarily lay very nearly in a straight line. A curve having been passed through these points, its abscissas were measured at the places whose ordinates corresponded to each half-revolution of the micrometer-screw, and from these measures the required wave-lengths were readily computed. For the ordinates the scale employed was one inch to each revolution of the micrometer-screw, and for the abscissas it was such that

$$\text{Log } \lambda = \frac{6.5 - \text{Log } x}{2}$$

in which λ is the wave-length, expressed in millionths of a millimeter, and x is the abscissa, expressed in inches. These scales were found quite sufficient to give the wave-lengths with as much accuracy as they can be observed in this instrument. The results are as follows:

[1] Nature, 1870, Vol. III, p. 104.

Table II.—*For converting Corrected Readings of the Spectroscope-Micrometer into Wave-Lengths.*

Corrected Micrometer Reading.	Wave-Length.	Difference.	Corrected Micrometer Reading.	Wave-Length.	Difference.
r.			r.		7.5
6.0	788.9	36.8	15.5	469.8	7.1
6.5	752.1	31.5	16.0	462.7	6.6
7.0	720.6	28.4	16.5	456.1	6.3
7.5	692.2	24.9	17.0	449.8	5.9
8.0	667.3	22.7	17.5	443.9	5.7
8.5	644.6	20.2	18.0	438.2	5.4
9.0	624.4	18.4	18.5	432.8	5.1
9.5	606.0	16.8	19.0	427.7	5.0
10.0	589.2	15.6	19.5	422.7	4.9
10.5	573.6	13.8	20.0	417.8	4.5
11.0	559.8	12.5	20.5	413.3	4.2
11.5	547.3	11.9	21.0	409.1	4.2
12.0	535.4	11.2	21.5	404.9	3.8
12.5	524.2	10.6	22.0	401.1	3.7
13.0	513.6	10.0	22.5	397.4	3.6
13.5	503.6	9.3	23.0	393.8	3.3
14.0	494.3	8.7	23.5	390.5	3.2
14.5	485.6	8.3	24.0	387.3	
15.0	477.3	7.5			

The diameter of the luminous disk of the micrometer is equal to 0.44 of a revolution of the screw, and it was always used as the unit when estimating distances in the spectrum. The opening of the slit was measured by holding a sodium-flame before it and comparing the width of the resulting bright band with the diameter of the micrometer-disk. When they were equal the jaws were 0.0125 of an inch apart, as may be readily computed from the constants given above.

II.—OBSERVATIONS OF ENCKE'S COMET.

Table III contains all the measures of the spectrum of the comet which were obtained at this Observatory. The micrometer readings were invariably made on the brightest part of each band.

Table III.

Date.	Observer.	Width of Slit.	Observed Micrometer Reading.			Mean of Observed Mic. Reading.		
			Sodium Flame.	First Band.	Second Band.	Sodium Flame.	First Band.	Second Band.
1871.		in.	r.	r.	r.	r.	r.	r.
Nov. 18	Harkness . .	0.005	13.55	. .	17.3			
			.52	. .	.3			
			.52	. .	.2	13.53	. .	17.27
18	Hall . . .	. .	13.62	. .	17.2			
			.62	. .	.25			
			.60	. .	.2	13.61	. .	17.22
25	Harkness . .	0.020	10.16	. .	13.65			
					.55			
					.53			
					.71			
			10.16	. .	.45	10.16	. .	13.58
25	Hall . . .	. .	. .	. .	13.63			
					.48			
					.40	. .	. .	13.50
26	Harkness . .	0.020	10.38	. .	13.95			
			.40	. .	.68			
			.42	. .	.80	10.40	. .	13.81
27	Harkness . .	0.022	10.42	. .	13.65			
			.44	. .	.67			
			.42	. .	.60			
			.43	. .	.62			
			.44	. .	.62	10.43	. .	13.63
29	Harkness . .	0.020	10.51	. .	13.68			
			.50	. .	.68			
			.50	. .	.70			
			.50	. .	.77			
			.50	12.3	.69	10.50	12.3	13.70
Dec. 1	Harkness . .	0.017	10.51	. .	13.57			
			.51	. .	.70			
			.51	11.95	.70			
			.49	.99	.62			
			.48	.80	.64	10.50	11.91	13.65
2	Harkness . .	0.020	10.50	. .	13.60			
			.50	. .	.55			
			.50	11.88	.68			
			.45	.95	.52			
			.49	.92	.61	10.49	11.92	13.59

The following are the notes which accompanied these observations:

November 18, 1871.—A fine moonless evening. Found the spectrum of Encke's Comet to consist of two bright bands, in each of which the light was most intense in the middle of its breadth, and shaded off toward the edges. The more refrangible band was certainly three or four times brighter than the other, and was about equal in breadth to the diameter of the micrometer-disk. The fainter band was, perhaps, a little narrower. No continuous spectrum was visible.

November 25.—Moon nearly full and atmosphere a little hazy; consequently, the spectrum of the comet was very difficult to observe, the bright band being excessively faint, and the other one not sufficiently visible to enable me to estimate its position satisfactorily. As the slit was gradually closed the bands slowly faded away without getting perceptibly narrower. In the large equatorial of $9\frac{1}{2}$ inches aperture the comet appeared as a faint nebulous cloud, about four or five minutes in diameter, with a slight condensation at its center, but no trace of a nucleus.

November 26.—Moon nearly full, and the sky, in addition to being quite hazy, was covered with drifting clouds, between which the observations were made. The comet was very faint indeed, and its spectrum so dim that the less refrangible band was only visible by glimpses. Both last night and to-night I have seen a faint continuous spectrum, but it appears to be due to the moonlight, for it was found in all parts of the sky to which the telescope was pointed.

November 27.—A fine, clear evening, with the full moon about ten degrees high. The distance of the less refrangible band of the spectrum from the other was estimated to be two and a half times the diameter of the micrometer-disk. This makes its reading $13^{r}.63 - 2.5 \times 0^{r}.44 = 12^{r}.53$. The more refrangible band is certainly three times as bright as the other. The faint continuous spectrum from the moonlight in the sky is still visible.

November 29.—The evening was very clear and the observations were finished before the moon rose. The breadth of each of the bands composing the spectrum was the same, and about equal to the diameter of the micrometer-disk. The more refrangible band was certainly three or four times brighter than the other. No trace of a continuous spectrum was detected.

December 1.—A fine, clear, moonless evening. The comet was brighter than ever before, and just barely visible to the naked eye. In the $9\frac{1}{2}$-inch equatorial it showed considerable condensation, which, however, was not central, but mostly on the side farthest from the sun. Upon a careful examination of the spectrum in an absolutely dark field, I discovered a third band, which I estimated to be more refrangible than either of the other two by about twice the interval between them. This makes its micrometer reading $13^{r}.65 + 2 \times 1^{r}.74 = 17^{r}.1$. If we designate the least refrangible as the first, the next as the second, and the most refrangible as the third; then calling the brightness of the third band unity, that of the second was approximately sixteen, and that of the first four. The brightest part of the first and second bands was altogether on the less refrangible side of their centers, and was about equal in breadth to the diameter of the micrometer-disk; while the total breadth of each band was about twice the diameter of that disk. The third band was of nearly uniform

brightness throughout its whole breadth, which was also about twice the diameter of the micrometer-disk. Upon narrowing the opening of the slit, the bands did not become lines, but gradually faded out. I looked very carefully for a continuous spectrum, but am in doubt whether or not one existed.

I examined the comet closely for polarization, employing a comet-seeker of four inches aperture and a double-image prism, but no change in the relative brightness of the two images was perceptible on rotating the prism.

December 2.—A fine, clear, moonless evening, but the comet had an altitude of only twenty degrees and seemed scarcely so bright as it was yesterday. The third band in its spectrum was barely visible, and I did not even attempt to estimate its position. During all my observations of this comet I have fancied, at times, that it gave a faint continuous spectrum, but, upon looking carefully for it, I never could feel certain of its existence.

Table IV contains the final numerical results of the observations on the spectrum of the comet. The corrected micrometer readings were obtained by subtracting from the observed, or estimated, readings the quantity necessary to reduce the mean of the readings on the sodium-flame to 10.00 revolutions, as already explained. The wave-lengths are expressed in millionths of a millimeter, and have been taken from Table II, with the argument "Corrected micrometer reading."

TABLE IV.

Date.	Corrected Micrometer Reading.			Wave-Length.		
	First Band.	Second Band.	Third Band.	First Band.	Second Band.	Third Band.
1871.	r.	r.	r.			
Nov. 18	. .	13.68	. .	. .	500.2	
25	. .	.38	. .	. .	505.9	
26	. .	.41	. .	. .	505.4	
27	12.10	.20	. .	533.	509.6	
29	11.8	.20	. .	540.	509.6	
Dec. 1	.41	.15	16.6	549.5	510.6	455.
2	.43	.10	. .	549.0	511.6	

Perhaps the most remarkable feature about this series of observations is the continual increase in the wave-length of the light emitted by the brightest part of the second band of the spectrum. If the amount of the change had been small it might have been attributed to accidental errors in pointing the micrometer, but it is much too great to admit of such an explanation. Zöllner has shown that "the ratio of brightness of two adjacent places in the spectrum (of a gas) may be reversed by alterations of temperature, and a minimum appear in the place of a former maximum."[1] This is probably the key to the change in question. The comet was approaching the sun.

[1] Zöllner on the Influence of Density and Temperature on the Spectra of Incandescent Gases. L. E. & D. Phil. Mag., 1871, Vol. 41, p. 199.

Its temperature must, therefore, have been increasing; and, in consequence of that increase, the character of its light was slightly modified. So far as I am aware, this is the first time that such a phenomenon has been observed in a celestial body.

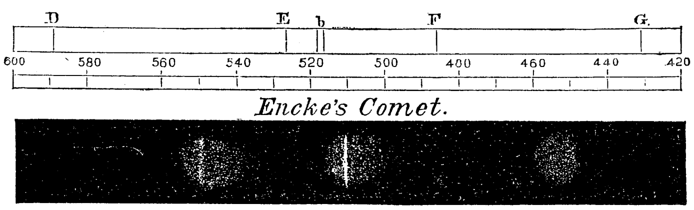
The comet was decidedly best seen on the evening of December 1; and, by the joint use of the micrometer readings and notes made on that occasion, Table V has been constructed. It gives the numerical data for the most important features of the spectrum, but it must be remembered that the positions of the *edges* of the bands were only estimated.

TABLE V.

	Encke's Comet.			Comet II, 1868.
	Relative Brightness.	Corrected Micrometer Reading.	Wave-Length.	Wave-Length.
		r.		
First Band	4			
Less Refrangible Edge	.	11.19	555.	562.
Brightest Part	.	11.41	549.5	
More Refrangible Edge	.	12.07	533.	542.
Second Band	16			
Less Refrangible Edge	.	12.93	515.	516.
Brightest Part	.	13.15	510.6	
More Refrangible Edge	.	13.81	497.	499.
Third Band	1			
Less Refrangible Edge	.	16.16	461.	471.
Middle	.	16.6	455.	
More Refrangible Edge	.	17.04	449	459.

Fig. 2 conveys a very good idea of the general appearance of the spectrum, and, in addition, shows the position of the bright bands relatively to some of the most conspicuous lines in the solar spectrum.

Fig. 2.



Mr. Huggins has published a careful description of the spectrum of Comet II, 1868,[1] but, as he has not furnished the means of converting his micrometer readings

[1] Further Observations on the Spectra of some of the Stars and Nebulæ, with an Attempt to determine therefrom whether these Bodies are moving toward or from the Earth; also Observations on the Spectra of the Sun and of Comet II, 1868. Phil. Trans., 1868, p. 555.

into any known scale, in order to obtain the wave-lengths it is necessary to have recourse to the plate accompanying his paper. In addition to the spectrum of the comet, this exhibits also some of the principal lines of the solar spectrum, and by interpolating between them the numbers contained in the last column of Table V have been deduced. A comparison of these numbers with those contained in the preceding column of the same table will show that the wave-lengths of the spectrum of Encke's Comet are so nearly identical with those of Comet II, 1868, as to render it almost certain that the differences between them are entirely due to the far from accurate methods employed in their determination. When, in addition to this, the great similarity in the general appearance of the two spectra is considered, it seems impossible to avoid concluding that they are absolutely identical. Hence it follows that the physical constitutions of the two comets are also identical; and, as Mr. Huggins has shown that Comet II, 1868, is probably composed mainly of incandescent carbon in a gaseous state, we infer that Encke's comet is also composed of the same substance. At all events, my observations show that it gives a carbon-spectrum exactly like that obtained from an electric spark taken in olefiant gas.

III.—DENSITY OF ENCKE'S SUPPOSED RESISTING MEDIUM IN SPACE.

As it was from a consideration of the motion of this comet that Encke was led in 1819 to propound his famous theory of a resisting medium in space, perhaps it may not be inappropriate to incorporate in this report an attempt to determine the density of that medium, but in so doing I do not wish to be understood as expressing any opinion with regard to the truth or falsity of Encke's hypothesis. The problem to be considered may be stated as follows:

Supposing all space to be pervaded by a resisting medium of uniform density, what must that density be in order to account for the observed retardation of Encke's Comet?

1.—*Investigation of a General Expression for the Mass of a Comet.*

If we assume the sun's horizontal equatorial parallax to be 8″.9, and adopt Captain Clarke's elements of the figure of the earth,[1] namely,

Equatorial semi-diameter	3963.057 miles,
Polar semi-diameter	3949.760 miles,

we shall have, for the distance of the earth from the sun,

$$\frac{3963.06}{\sin 8''.9} = 91\ 847\ 000 \text{ miles.}$$

[1] Phil. Trans., 1856, p. 626.

The following are the principal results which have hitherto been obtained for the average specific gravity of the earth, water being taken as unity:

Schehallien experiment[1]	5.0
Cavendish's experiment, reduced by Baily[2]	5.45
Reich, by Cavendish's method	5.44
Baily, by Cavendish's method[3]	5.67
Airy, by his Harton Colliery experiment[4]	6.565
Captain Clarke, from deflections of the plumb-line at Arthur's Seat[5]	5.316

The mean of all is 5.57, which I shall use as the best value at present attainable.

The following notation will be employed:

Δ = mean distance of the earth from the sun = 91 847 000 English miles.

ρ'' = distance of the comet from the earth, expressed in terms of the earth's mean radius-vector.

s = apparent semi-diameter of the comet as seen from the earth, expressed in arc.

m = mass of comet.

σ = specific gravity of the gas composing the comet, taken at its actual pressure and temperature, which are supposed to be t degrees F. and p inches of mercury.

σ_0 = specific gravity of the gas composing the comet, taken at a temperature of 60° F., and a pressure of 30 inches of mercury at 60° F., the specific gravity of air being considered as unity.

π = ratio of the circumference of a circle to its diameter = 3.14159.

w = weight of one cubic foot of air at a temperature of 60° F. and a pressure of 30 inches of mercury = 534.55 grains.

f = number of feet in one English mile = 5280.

γ = number of grains in one avoirdupois pound = 7000.

β = number of pounds in one ton = 2000.

If $V_{(p,t)}$ is the volume, and σ the specific gravity, of a certain quantity of gas at the pressure p and temperature t, and $V_{(p',t')}$ its volume, and σ' its specific gravity, at the pressure p' and temperature t', then

$$\sigma V_{(p,t)} = \sigma' V_{(p',t')}$$

and, also,

$$V_{(p,t)} = V_{(p',t')} \frac{(1 + \alpha t) p'}{(1 + \alpha t') p}$$

where α is the coefficient of expansion, which is nearly the same for all gases. Hence

$$\sigma = \sigma' \frac{(1 + \alpha t') p}{(1 + \alpha t) p'}$$

[1] Abridg. Phil. Trans., 1776-80, Vol. XIV, p. 426.
[2] Mem. Roy. Ast. Soc., Vol. XIV, p. 91.
[3] *Ibid.*, p. ccxlvii.
[4] Phil. Trans., 1856, p. 342.
[5] *Ibid.*, p. 606.

Employing the Fahrenheit scale of temperatures, for which $\alpha = \frac{1}{459}$, and putting $p' =$ 30 inches, and $t' = 60°$, this becomes

$$\sigma = \sigma_0 \frac{519\,p}{(459 + t)\,30} \qquad (1)$$

by means of which the specific gravity of a comet may be computed, provided the gas of which it is composed, and also its pressure and temperature, can be discovered.

If the distance of a comet from the sun is $\Delta \rho''$ miles, its radius will be $\Delta \rho''$ sin s miles, and its volume will be $\frac{4}{3}\pi(\Delta\rho'' \sin s)^3$ cubic miles. One cubic foot of its substance will weigh σw grains, and one cubic mile will weigh $\sigma w f^3 \div \beta\gamma$ tons. The total mass of the comet will, therefore, be

$$\frac{\frac{4}{3}\pi\, w f^3 \Delta^3}{\beta\gamma}\,\sigma(\rho'' \sin s)^3 = 1\,824 \times 10^{28}\,\sigma(\rho'' \sin s)^3 \text{ tons.} \qquad (2)$$

Taking the mean radius of the earth as 3956.4 miles, its volume is $\frac{4}{3}\pi\; 3956.4^3$ cubic miles. As already shown, its density is 5.57 times that of water, which weighs 62.50 pounds per cubic foot. Hence, the total weight of the earth is $\frac{4}{3}\pi \times 5.57 \times 62.50 \times 3956.4^3 f^3 \div \beta = 6\,647 \times 10^{18}$ tons; and the mass of the comet, expressed in terms of the earth's mass as unity, will be

$$\frac{1\,824 \times 10^{28}}{6\,647 \times 10^{18}}\,\sigma(\rho'' \sin s)^3 = 2\,744 \times 10^6\,\sigma(\rho'' \sin s)^3 \qquad (3)$$

Lead weighs 709.5 pounds per cubic foot; hence, a sphere of that metal having a radius of L miles will weigh $\frac{4}{3}\pi \times 709.5\, f^3 L^3 \div \beta = 2\,187 \times 10^{18} L^3$ tons. Equating this with the right-hand member of equation (2) we find that the weight of the comet will be equivalent to that of a sphere of lead having a radius of

$$4\,369\,000\,\sqrt[3]{\sigma}\,(\rho'' \sin s) \text{ miles.} \qquad (4)$$

2.—*Investigation of a General Expression for the Amount of Free Force developed by the shortening from a to a′ of the Semi-axis Major of a Body moving about the Sun in a Closed Orbit.*

In addition to the notation already given, the following will be required:

V = velocity of the body in its orbit at the time its radius-vector is ρ'.

a, a' = semi-axes major at the times T and T', expressed in terms of the earth's mean radius-vector as unity.

ρ, ρ' = radii-vectores at the times T and T'.

v = true anomaly.

k = a constant peculiar to the solar system. Log k = 8.23558.

g = mean force of gravity at the surface of the earth, expressed in terms of the velocity which a body falling freely would acquire at the end of the first second = 32.180 feet.

If a body whose mass is insignificant compared with that of the sun move about the latter in a closed orbit, its velocity, at any time, will be given by the equation

$$V^2 = k^2 \left(\frac{2}{\rho'} - \frac{1}{a} \right)$$

As the units in this expression are the earth's mean radius-vector and the mean solar day, in order to obtain V in feet per second the right-hand member of the equation must be multiplied by the square of the number of feet in the earth's mean distance from the sun, and divided by the square of the number of seconds in a day. We, therefore, have

$$V^2 = \left(\frac{2}{\rho'} - \frac{1}{a} \right) \frac{k^2 \Delta^2 f^2}{86\,400^2} = 9\,322 \times 10^6 \left(\frac{2}{\rho'} - \frac{1}{a} \right) \tag{5}$$

Imagining the sun's whole mass to be concentrated at its center, we may suppose the orbit of the body to become more and more eccentric, until finally it approaches indefinitely near a straight line, having the foci at its two extremities. Under such circumstances it is evident that when the body attained its aphelion it would be, for an instant, in a state of absolute rest, and then it would begin its return to the sun, which it would approach with ever-increasing velocity until it arrived at perihelion, when, for an instant, its velocity would be infinite; and then it would again recede, with diminishing velocity, to its aphelion, and thus continue to oscillate back and forth for evermore. If the aphelion distance is designated by ρ, the mean distance will evidently be $\frac{1}{2}\rho$, and we shall have

$$V^2 = 9\,322 \times 10^6 \left(\frac{2}{\rho'} - \frac{2}{\rho} \right)$$

A moment's consideration will show that this expression applies also to the case of a body situated at a distance ρ from the sun, and falling toward it, from a state of rest, through the distance $\rho - \rho'$. But the work done in imparting a velocity of V feet per second to a body having a mass of m pounds is

$$\frac{m\,V^2}{2\,g} \text{ foot-pounds.}$$

Hence, the amount of energy which the sun would impart to a body while falling through the space of $\rho - \rho'$ is

$$1\,448 \times 10^5 \left(\frac{2}{\rho'} - \frac{2}{\rho} \right) m \text{ foot-pounds.} \tag{6}$$

Resuming equation (5), it is evident that if at the epoch T we suppose the body to have had the true anomaly v, the semi-axis major a, and the radius-vector ρ, its velocity in its orbit was

$$\left\{ 9\,322 \times 10^6 \left(\frac{2}{\rho} - \frac{1}{a} \right) \right\}^{\frac{1}{2}} \text{ feet per second,}$$

and the work required to produce this velocity was

$$1\,448 \times 10^5 \left(\frac{2}{\rho} - \frac{1}{a} \right) m \text{ foot-pounds.}$$

Again, if at the epoch T', the eccentricity of the orbit and the true anomaly being the same as before, we suppose the body to have had the semi-axis major a', and the radius-vector ρ', the work required to produce the velocity which it then had in its orbit was

$$1\,448 \times 10^5 \left(\frac{2}{\rho'} - \frac{1}{a'} \right) m \text{ foot-pounds.}$$

Hence, while the radius-vector shortened from ρ to ρ', a force of

$$1\,448 \times 10^5 \left(\frac{2}{\rho'} - \frac{2}{\rho} + \frac{1}{a} - \frac{1}{a'} \right) m \text{ foot-pounds}$$

was expended in increasing the velocity of the body's motion. Subtracting this from the total amount of energy imparted by the sun's attraction during the same time, as given by equation (6), there remains a free force of

$$1\,448 \times 10^5 \left(\frac{1}{a'} - \frac{1}{a} \right) m \text{ foot-pounds,} \qquad (7)$$

which may have been applied to overcoming the resistance of a medium pervading space.

3.—*Investigation of a General Expression for the Density of a Medium, in Terms of the Resistance experienced by a Body of Known Size moving through it with known Velocity.*

It is a very long step from the speed of a cannon-ball to that of a planet or comet; and yet, as the highest velocities with which we have hitherto been able to experiment are furnished by projectiles, in an investigation like the present we have no recourse except to argue from the resistance which they experience to that which may be experienced by bodies moving about the sun.

The law connecting the velocity of a projectile with the resistance offered to it by the air does not seem to be well determined, but perhaps the most satisfactory formula is that of Mr. J. A. Longridge,[1] who gives

$$\text{Log } R = 4.5 \log V - 13.0155$$

where R is the resistance per square inch of sectional area, expressed in avoirdupois

[1] Proceedings Royal Society, 1868, Vol. XVI, p. 265; or, L. E. & D. Phil. Mag., 1868, Vol. XXXV, p. 305.

pounds, and V is the velocity in feet per second. If R is taken in tons per square mile, the formula will become

$$\text{Log } R = 4.5 \log V - 6.7129$$

that is,

$$R = \frac{V^{4.5}}{5\,163\,000}$$

which corresponds to an atmospheric pressure of about thirty inches of mercury. If the sectional area of the projectile is α square miles, and the resistance is supposed to vary as the pressure, then, for a pressure of p inches, the total resistance will be

$$R = \frac{\alpha\, p\, V^{4.5}}{154\,900\,000}$$

Hence, if we know the velocity of a body, its sectional area, and the resistance with which it meets, we can compute approximately the density of the medium through which it is moving by means of the formula

$$p = \frac{154\,900\,000\; R}{\alpha\, V^{4.5}} \tag{8}$$

4.—*Investigation of a General Expression for the Density of the Supposed Resisting Medium in Space.*

If a body moving about the sun in a closed orbit passes over a distance of F feet between the epochs T and T', and if, during that period, its semi-major axis is shortened from a to a', then it results from equation (7) that it must have experienced a constant resistance equal to

$$\frac{1\,448 \times 10^5}{F}\left(\frac{1}{a'} - \frac{1}{a}\right) m$$

Substituting for m its value from equation (2), this becomes

$$\frac{1\,448 \times 10^5}{F} \times \frac{\frac{4}{3}\, w f^3}{\beta\, \gamma}\, \sigma\, \pi\, (\Delta\, \rho'' \sin s)^3 \left(\frac{1}{a'} - \frac{1}{a}\right)$$

which is the value of R in equation (8). α, in the same equation, is equal to $\pi(\Delta\,\rho'' \sin s)^2$. Substituting these values of R and α, and reducing, equation (8) becomes

$$p = \frac{1\,544 \times 10^{28}\, \rho'' \sin s \left(\frac{1}{a'} - \frac{1}{a}\right) \sigma}{F\, V^{4.5}} \tag{9}$$

which is the formula required.

5.—*Application of the Equations found in Paragraphs* 1, 2, 3, *and* 4 *to the case of Encke's Comet.*

As already explained, the spectroscope shows that Encke's Comet is mainly, perhaps entirely, composed of carbon in a gaseous state, and that its spectrum is identical with that of olefiant gas, except that it lacks the hydrogen lines. But, owing to the failure of physicists in obtaining carbon vapor, no determination of its specific gravity exists, and, as a rough approximation, I shall therefore employ the specific gravity of olefiant gas, which, being a hydro-carbon, is probably lighter than the material composing the comet.

It is next necessary to have an estimate of the mean pressure and temperature of the gas which forms the comet. In judging of the first of these elements, there is scarcely anything to guide us, and I have somewhat arbitrarily assumed that it is not greater than 0.12 of an inch, nor less than 0.004 of an inch. As to the second element, Kirchhoff has shown that the temperature of a gaseous body, yielding a discontinuous spectrum of given wave-lengths, cannot be less than that of a perfectly black body whose spectrum is of the same brightness at the same wave-lengths, and this renders it certain that the temperature of the comet was not much less than 1 000° F. It is equally sure that it was not greater than that of the sun, which we will now investigate.

Secchi, from observations on radiation, estimates the temperature of the sun at 10 000 000° C.;[1] Ericsson, from observations of the same kind, made by means of a peculiar apparatus of his own, estimates it at 4 036 000° F. = 2 243 000 C.;[2] Zöllner, from his theory of the cause of the red prominences, estimates it at not less than 68 400° C.;[3] Lane, from the recorded effects of Parker's great burning-lens, estimates it at 55 450° F. = 30 788° C.;[4] and Spœrer estimates it at 27 000° C. Although these estimates differ widely from each other, they all seem to me extremely high, and probably vastly in excess of any temperature that ever existed in the universe. In the year 1838 Pouillet estimated the temperature of the sun as somewhere between 1 461° and 1 761° C.[5] Last January M. E. Vicaire, using Secchi's data, and reducing them by means of Dulong and Petit's formula for the intensity of radiation, found that the resulting temperature of the sun was 1 398° C., and, from an extended discussion of the subject, he concluded that the temperature of the solar surface is entirely comparable with that of terrestrial flames.[6] His paper was presented to the French Academy, and, after its reading, a number of the *savants* present expressed their substantial concurrence in his views, their estimates of the probable solar temperature all lying between 2 500° and 3 000° C.[7]

If R represents the total radiation in vacuo from a body whose temperature is $t + \theta$, situated in an inclosure whose temperature is θ, then, from a series of experiments, in which the value of $t + \theta$ ranged between 60° and 240° C., and that of θ was varied from 0° to 80° C., Dulong and Petit found that[8]

$$R = m\,\alpha^{\theta}(\alpha^{t} - 1)$$

[1] Le Soleil, p. 271.
[2] Nature, 1871, Vol. IV, p. 452.
[3] L. E. & D. Phil. Mag., 1870, Vol. XL, p. 323.
[4] Amer. Jour. of Science, 1870, Vol. L, p. 68.
[5] Comptes Rendus, 1838, T. VII, p. 35.
[6] *Ibid.*, 1872, T. LXXIV, pp. 33 and 34.
[7] *Ibid.*, pp. 35–36 and 152.
[8] Annales de Chimie et de Physique, 1817, T. VII, p. 252.

where α is a constant the numerical value of which for the centigrade scale of temperatures is 1.0077, and m is an unknown constant depending upon the unit of heat adopted, the size of the radiating body, the substance of which it is composed, and the time during which the radiation takes place. Assuming the truth of this equation, adopting as the unit of heat the quantity required to raise 1 000 grains of water 1° C., and making R the amount of radiant heat which emanates in one minute from a surface one foot square, Mr. Hopkins determined experimentally the approximate value of m for several different substances, as follows:[1]

Glass	9.566
Dry chalk	8.613
Dry new red sandstone	8.377
Sandstone, (building-stone)	8.882
Polished limestone	9.106
Unpolished limestone, (same block)	12.808

It is more convenient to take as the unit of heat the quantity required to raise one gram of water from 0° to 1° C., and to make R the radiation in one minute from a surface one centimeter square. As 1 000 grains are equal to 64.82 grams, and one square foot is equal to 929.0 square centimeters, in order to make the change it is only necessary to multiply the values given above by $\frac{64.82}{929.0} = 0.06978$. We then have

Glass	0.6674
Dry chalk	.6009
Dry new red sandstone	.5845
Sandstone, (building-stone)	.6197
Polished limestone	.6353
Unpolished limestone, (same block)	.8937

If r represents the intensity of solar radiation on a surface one centimeter square, situated at a distance of Δ centimeters from the sun, then the total solar radiation will be $4\pi\Delta^2 r$; and if the apparent angular semi-diameter of the sun as seen from the distance Δ is s, its radius will be $\Delta \sin s$, and its surface will be $4\pi\Delta^2 \sin^2 s$. Hence, the radiation at the solar surface from an area one centimeter square will be $\frac{r}{\sin^2 s}$, and, in accordance with Dulong and Petit's law, we shall have

$$\frac{r}{\sin^2 s} = m\,\alpha^\theta(\alpha^t - 1)$$

which at once gives

$$t + \theta = \frac{\log\left(\alpha^\theta + \dfrac{r}{m \sin^2 s}\right)}{\log \alpha} \qquad (10)$$

If, in this formula, while m and α retain the same signification as before, we make θ represent the temperature of space beyond the limit of the earth's atmosphere, r the

[1] Phil. Trans., 1860, p. 407.

intensity of solar radiation at the earth's surface, corrected for atmospheric absorption, and s the sun's semi-diameter at the earth's mean distance, then $t + \theta$ will be the temperature of the sun.[1]

Probably the best determinations of the temperature of space are those of Pouillet and Sir J. F. W. Herschel. The former gives $-142°$ C.,[2] and the latter $-239°$ F. $= -151°$ C.[3] The mean is $-146°$ C., which I adopt as the value of θ. Taking as the unit of heat the quantity required to raise one gram of water from 0° to 1° C., Pouillet found, from observations made at Paris, that if the atmosphere did not exist, the sun's rays, falling vertically on a surface one centimeter square, would be capable of imparting to it 1.7633 units of heat per minute.[4] Sir J. F. W. Herschel found, from observations made at the Cape of Good Hope, that when the sun was within 12° of the zenith its rays, after passing through the earth's atmosphere, were capable of melting 0.1914 of a millimeter of ice per minute.[5] Employing the value used by him for the latent heat of ice, viz, 135°.5 F. = 75°.28 C., and, taking the atmospheric absorption within 12° of the zenith to be 0.25, as given by Pouillet, this becomes 1.801 units of heat for each square centimeter of surface. The mean between this result and that of Pouillet is 1.782 units, which I adopt as the value of r. The value of s is 16′ 2″. Mr. Hopkins's experiments seem to indicate that for the substances composing the earth's crust the value of m is about 0.67, but, as we have no knowledge of its value for the sun, I shall assume that it lies somewhere between 0.1 and unity.

Collecting our results, we have

$$\theta = -146° \text{ C.} \qquad r = 1.782 \qquad s = 16' \ 2''$$
m lies somewhere between 0.1 and 1.0

Substituting these values in equation (10) we obtain

m	$t + \theta$	m	$t + \theta$
0.1	1775° C.	0.6	1542° C.
0.2	1685	0.7	1522
0.3	1632	0.8	1504
0.4	1594	0.9	1489
0.5	1565	1.0	1475

In order to make $t + \theta$ as great as 2 000° C. we must assume m as small as 0.018; and, if Dulong and Petit's formula holds for such elevated temperatures, it does not seem possible that the sun can be hotter than the oxyhydrogen flame, the temperature of which, according to Bunsen, is about 2 844° C.[6]

[1] This equation is essentially the same as that given by Pouillet in the Comptes Rendus, 1838, T. VII, p. 35; but, as that work is not generally accessible in this country, and as the above was written before I had seen Pouillet's memoir, it is allowed to remain in this report.

[2] Comptes Rendus, 1838, T. VII, p. 61.

[3] Encyclopædia Britannica, 8th edition, Art. Meteorology, par. 36.

[4] Comptes Rendus, 1838, T. VII, p. 31.

[5] Results of Astronomical Observations made at the Cape of Good Hope, by Sir J. F. W. Herschel, p. 446.

[6] Poggendorf's Annalen, Vol. CXXXI, p. 172.

Returning to the consideration of the comet, in view of what has just been said, it is very unlikely that its temperature approached 5 000° F.; but, in order to be on the safe side, I will assume that t was not less than 1 000°, nor greater than 20 000° F. Collecting our results, we have $\sigma_0 = 0.9784$, p comprised between the limits 0.004 and 0.12 of an inch of mercury, and t comprised between the limits 1 000° and 20 000° F. Substituting these values in equation (1), we obtain the values of log σ given in

TABLE VI.

NOTE.—In using these logarithms their characteristics must be diminished by 10.

Pressure in Inches.	Temperature.			
	1 000°	5 000°	10 000°	20 000°
0.004	5.6666	5.0935	4.8111	4.5197
.04	6.6666	6.0935	5.8111	5.5197
.08	6.9676	6.3946	6.1122	5.8208
.12	7.1437	6.5706	6.2883	5.9969

On the 25th of November the apparent diameter of the comet was from four to five minutes. In order to be on the safe side I will take $s = 2'$. For that date Mr. S. von Glasenapp's ephemeris[1] gives log $\rho'' = 9.5170$. Hence, log $(\rho'' \sin s) = 6.2818 - 10$. Substituting that value in equation (3), it becomes $0.01922\ \sigma$, by means of which Table VII has been computed.

TABLE VII.—*Mass of Encke's Comet in terms of the Earth's Mass as Unity.*

Pressure in Inches.	Temperature.			
	1 000°	5 000°	10 000°	20 000°
0.004	0.000 000 892	0.000 000 238	0.000 000 124	0.000 000 064
.04	.000 008 920	.000 002 384	.000 001 244	.000 000 636
.08	.000 017 84	.000 004 768	.000 002 489	.000 001 273
.12	.000 026 76	.000 007 152	.000 003 733	.000 001 909

Substituting in equation (4) the value of $\rho'' \sin s$, given above, it becomes $836.0\sqrt[3]{\sigma}$, and the formula for the diameter in miles of a sphere of lead having the same mass as the comet is $1\,672\sqrt[3]{\sigma}$, by means of which Table VIII has been computed.

[1] Astronomische Nachrichten, No. 1854, Band 78, p. 90.

TABLE VIII.—*Diameter in Miles of a Sphere of Lead whose Mass is equal to that of Encke's Comet.*

Pressure in Inches.	Temperature.			
	1 000°	5 000°	10 000°	20 000°
0.004	60.1	38.70	31.16	24.91
.04	129.4	83.37	67.12	53.67
.08	163.1	105.05	84.58	67.63
.12	186.6	120.26	96.83	77.42

The extreme levity of comets has been so strongly insisted upon that the numbers in Tables VII and VIII seem, at first sight, surprisingly large, and perhaps many persons will feel inclined to doubt their accuracy when they remember that on August 23, 1779, Lexell's comet passed nearer to the planet Jupiter than its fourth satellite, and, although none of the satellites suffered any perceptible perturbation, the orbit of the comet was completely changed. However, that fact is entirely consistent with the numbers given in the Tables, for the mass of Jupiter's smallest satellite is 0.005 868, while the largest mass deduced for the comet is only 0.000 027; or, in other words, the mass of the satellite is at least 219 times greater than that of the comet, and no perturbations visible from the earth were to be expected.

The mean density of the earth is 2.037 times less than that of lead. Hence, multiplying the numbers in Table VIII by $\sqrt[3]{2.037} = 1.268$, we find that the mass of the comet was approximately equal to that of a sphere having the mean density of the earth and a diameter somewhere between 32 and 237 miles. Mr. Stone has shown that the diameters of the first seventy-one asteroids are probably all included between the limits 17 and 214 miles.[1] Their density is unknown, but it is not likely to exceed that of the earth. We are therefore led to conclude that *the mass of Encke's Comet is not less than that of an asteroid*, and this conclusion cannot be modified by any probable change in the adopted value of σ_0; for, even if we suppose the comet to consist of pure hydrogen, existing at the pressures and temperatures given in Table VIII, its mass must have been equal to that of a sphere having the mean density of the earth and a diameter somewhere between 13 and 98 miles. Few persons realize that gases are heavy, and yet one cubic mile of atmospheric air weighs 5 621 000 tons; and one cubic mile of pure hydrogen, the lightest known substance, weighs no less than 389 000 tons.

In order to obtain numerical values of a, a', F, and $V^{4.5}$, in equation (9), it is necessary to have the elements of the orbit of Encke's Comet, at two epochs, as widely

[1] Approximate Relative Dimensions of Seventy-one of the Asteroids, by E. J. Stone. Monthly Notices; Roy. Ast. Soc., 1867, Vol. XXVII, p. 302.

separated as possible. For the first epoch, I will take the perihelion passage of 1819, January 27.2934, Berlin mean time, when, according to Encke,[1]

$$
\begin{aligned}
e &= 0.84858 \\
\log q &= 9.52538 \\
\log a &= 0.34520
\end{aligned}
$$

and, for the second epoch, I will take the perihelion passage of 1871, December 28.6297, Berlin mean time, when, according to Mr. S. von Glasenapp,[2]

$$
\begin{aligned}
e' &= 0.84936 \\
\log q' &= 9.52229 \\
\log a' &= 0.34433
\end{aligned}
$$

Taking the mean of these two sets of elements, we obtain

$$
\begin{aligned}
e_0 &= 0.84897 \\
\log q_0 &= 9.52384
\end{aligned}
$$

Whence, by means of the formulæ,

$$a = \frac{q}{1 - e}$$

$$b = a(1 - e^2)^{\frac{1}{2}}$$

$$\mu = \frac{3548''.2}{a^{\frac{3}{2}}}$$

$$\text{Periodic Time} = \frac{360^\circ}{\mu}$$

we obtain

$$
\begin{aligned}
\log a_0 &= 0.34478 \\
\log b_0 &= 0.06778 \\
\mu_0 &= 1078''.55 \\
\text{Periodic Time} &= 1201.6 \text{ days} = 3.2899 \text{ years.}
\end{aligned}
$$

During the interval between the adopted epochs, the mean length of the circumference of the comet's orbit was approximately

$$2\pi a_0(1 - 0.25\,e_0^2 - 0.046\,875\,e_0^4 - 0.019\,532\,e_0^6)$$

the numerical value of which is 10.95 times the earth's mean radius-vector. But, between January, 1819, and December, 1871, the comet made sixteen complete revo-

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[1] Olbers Abhandlung, Encke's edition, p. 184. [2] Astronomische Nachrichten, No. 1854, Band 78, p. 90.

lutions, and therefore traveled $16 \times 10.95 = 175.2$ mean terrestrial radii-vectores, or $8\,498 \times 10^{10}$ feet, which is the value of F in equation (9).

Let

T = time of perihelion passage,
e = eccentricity,
q = perihelion distance,
a = semi-major axis,
b = semi-minor axis,
μ = mean daily motion,
M = mean anomaly at the time t,
E = eccentric anomaly at the time t,
v = true anomaly at the time t,
ρ = radius-vector at the time t,
V = velocity in the orbit, in feet per second, at the time t;

then we have the well-known formulæ

$$\mu(t - T) = M$$

$$M = E - e \sin E$$

$$\tan \tfrac{1}{2}v = \sqrt{\frac{1 + e}{1 - e}} \tan \tfrac{1}{2}E$$

$$\rho = a(1 - e)\left(\frac{\cos \frac{1}{2}E}{\cos \frac{1}{2}v}\right)$$

and also equation (5), which is

$$V^2 = 9\,322 \times 10^6\left(\frac{2}{\rho} - \frac{1}{a}\right)$$

by means of which the logarithmic values of $V^{4.5}$ have been computed from the elements given above, as follows:

	d.		d.	d.	d.	d.	d.	d.	d.	d.
$t - T$	0.00	20.03	40.05	60.08	120.16	180.24	240.32	360.48	480.64	600.80
Log $(t - T)$	. . .	1.30168	1.60260	1.77873	2.07976	2.25585	2.38079	2.55688	2.68182	2.77873
Log μ	. . .	3.03284								
Log M	. . .	4.33452	4.63544	4.81157	5.11260	5.28869	5.41363	5.58972	5.71466	5.81157
M	0° 0′ 0″	6° 0′ 3″	11° 59′ 56″	17° 59′ 59″	36° 0′ 0″	54° 0′ 0″	72° 0′ 0″	108° 0′ 0″	144° 0′ 0″	180° 0′ 0″
E	0° 0′ 0″	31° 11′ 50″	48° 20′ 30″	60° 13′ 0″	84° 24′ 38″	101° 38′ 30″	115° 47′ 44″	139° 33′ 18″	160° 21′ 15″	180° 0′ 0″
$e \sin E$	. . .	25° 11′ 47″	36° 20′ 30″	42° 13′ 0″	48° 24′ 40″	47° 38′ 30″	43° 47′ 40″	31° 33′ 20″	16° 21′ 14″	
$\frac{1}{2} E$	0° 0′ 0″	15° 35′ 55″	24° 10′ 15″	30° 6′ 30″	42° 12′ 19″	50° 49′ 15″	57° 53′ 52″	69° 46′ 39″	80° 10′ 37″	90° 0′ 0″
Log tan $\frac{1}{2} E$	. . .	9.44588	9.65206	9.76333	9.95757	0.08885	0.20248	0.43371	0.76158	
Log $\sqrt{\frac{1+e}{1-e}}$	. . .	0.54394								
Log tan $\frac{1}{2} v$	. . .	9.98982	0.19600	0.30727	0.50151	0.63279	0.74642	0.97765	1.30552	
$\frac{1}{2} v$	0° 0′ 0″	44° 19′ 45″	57° 30′ 40″	63° 45′ 46″	72° 30′ 31″	76° 53′ 19″	79° 50′ 8″	83° 59′ 24″	87° 10′ 1″	90° 0′ 0″
Log cos $\frac{1}{2} E$	. . .	9.98370	9.96015	9.93706	9.86967	9.80054	9.72544	9.53865	9.23199	
Log cos $\frac{1}{2} v$	. . .	9.85451	9.73008	9.64551	9.47794	9.35573	9.24668	9.01995	8.69396	
Log $\left(\frac{\cos \frac{1}{2} E}{\cos \frac{1}{2} v}\right)$	. . .	0.12919	0.23007	0.29155	0.39173	0.44481	0.47876	0.51870	0.53803	
Log $\left(\frac{\cos \frac{1}{2} E}{\cos \frac{1}{2} v}\right)^2$	. . .	0.25838	0.46014	0.58310	0.78346	0.88962	0.95752	1.03740	1.07606	
Log $a(1 - e)$	. . .	9.52384								
Log ρ	9.52384	9.78222	9.98398	0.10694	0.30730	0.41346	0.48136	0.56124	0.59990	0.61171
Log $\frac{2}{\rho}$	0.77719	0.51881	0.31705	0.19409	9.99373	9.88757	9.81967	9.73979	9.70113	9.68932
$\frac{2}{\rho}$	5.9867	3.3022	2.0752	1.5635	0.98567	0.77192	0.66019	0.54928	0.50249	0.48901
$\frac{1}{a}$	0.4521	. . .	. . .	. . .	0.45208					
$\frac{2}{\rho} - \frac{1}{a}$	5.5346	2.8501	1.6231	1.1114	0.53359	0.31984	0.20811	0.09720	0.05041	0.03693
Log $\left(\frac{2}{\rho} - \frac{1}{a}\right)$	0.7431	0.4549	0.2103	0.0459	9.7272	9.5049	9.3183	8.9877	8.7025	8.5674
Log (9322×10^6)	9.9695									
Log V^2	10.7126	10.4244	10.1798	10.0154	9.6967	9.4744	9.2878	8.9572	8.6720	8.5369
Log $V^{4.5}$	24.1033	23.4549	22.9045	22.5346	21.8176	21.3174	20.8975	20.1537	19.5120	19.2080

The second column of the following table contains the values of $V^{4.5}$, corresponding to the values of $t - T$, given in the first column. The third column contains a series of factors, so taken that if each value of $V^{4.5}$ is multiplied by the factor standing on the same line with it, the sum of all the products thus formed is approximately the mean value of $V^{4.5}$ for the whole orbit. The fourth column is sufficiently intelligible without further explanation.

$t - T$	$V^{4.5}$	Factor.	$V^{4.5} \times$ Factor.
0^d	$1\,268\,500 \times 10^{18}$	0.01667	$211\,400 \times 10^{17}$
20	285 000	.03333	95 020
40	80 260	.03333	26 750
60	34 250	.06667	22 830
120	6 571	.10000	6 571
180	2 077	.05000	1 039
240	790	.20000	1 580
360	142	.20000	285
480	33	.20000	65
600	16	.10000	16
			Sum $= 365\,556 \times 10^{17}$

Collecting our results, we have

$$\rho'' \sin s = 0.00019133 \qquad F = 8\,498 \times 10^{10}$$
$$a = 2.2141 \qquad V^{4.5} = 3\,656 \times 10^{19}$$
$$a' = 2.2097$$

Substituting these values in equation (9), it becomes

$$p = \frac{8\,612\,\sigma}{10^{16}}$$

by means of which Table IX has been computed, the values of σ being taken from Table VI.

TABLE IX.—*Density of the Supposed Resisting Medium in Space, expressed in terms of the Height in Inches of the Column of Mercury which it will support.*

[NOTE.—The numbers in the table are given in units of the fifteenth place of decimals.]

Pressure in Inches.	Temperature.			
	1 000°	5 000°	10 000°	20 000°
0.004	0.039 97	0.010 68	0.005 58	0.002 85
0.04	0.399 70	0.106 80	0.055 75	0.028 50
0.08	0.799 30	0.213 60	0.111 50	0.057 00
0.12	1.199 00	0.320 40	0.167 30	0.085 51